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MATRIX ANALYSIS METHODS FOR ANISOTROPIC INELASTIC STRUCTURES

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GRUMMAN AIRCRAFT ENGINEERING CORPORATION

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MATRIX ANALYSIS METHODS FOR ANISOTROPIC INELASTIC STRUCTURES

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FOREWORD

This report, prepared by the Grumman Aircraft Engineering Corporation, Bethpage, New York, covers work performed under Air Force Contract AF 33(615)-2260. The contract was sponsored by the Air Force Flight Dynamics Laboratory of the Research and Technology Division, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio. It was accomplished under Project No. 1467, "Structural Analysis Methods," Task 146701, "Stress-Strain Analysis Methods for Structures Exposed to Creep Environment." The work was administered by Mr. Laszlo Berke, FDTR, Project Engineer. The report covers work conducted from January 1965 to October 1965. The manuscript was released by the authors in February 1966 for publication as an RTD Technical Report.

The investigation was supervised by Dr. Warner Lansing, Chief of Structural Mechanics. The computer programming was carried out by Mr. Albert Davidson. Some preliminary elastic analysis data were automatically generated by the ASTRAL, Automated STRuctural AnaLysis program developed by Mr. Philip Mason.

The work of Dr. T. J. Mentel on the evaluation of procedures for carrying out inelastic analyses, and on the formulation of the biaxial stress procedure is also acknowledged. His "Instability Analysis of the Constant Stress and Constant Strain Methods" is reproduced from Reference 6 in Appendix III.

Acknowledgements are made to Drs. Gabriel Isakson and Harry Armen of the Grumman Research Department for their suggestions and review of certain portions of this work.

All computations were carried out at the Grumman Computing Center.

This technical report has been reviewed and is approved.

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ABSTRACT

Most aerospace structural materials exhibit some degree of anisotropic strain hardening. During the past few years, several methods have appeared in the literature for introducing inelastic isotropic material behavior effects into existing matrix analysis procedures using the incremental theory of plasticity. A review is presented of these methods and a step-by-step routine known as the "constant strain" method is selected for the development of an anisotropic inelastic procedure.

A simple truss with one redundant is used to indicate the basic ideas of the approach. Then the procedure is generalized to the more important case of biaxially stressed structures. Nodal stresses are evaluated step-wise for increasing load through the use of an influence coefficient equation. The inelastic (plastic and creep) strains at one load level are used as initial strains at the subsequent level to account for nonlinear effects. The anisotropic behavior is considered by using a proposed extension of Hu's strain hardening theory.

Several analyses of an aluminum alloy (2024-T4) shear lag structure, which has been tested previously for the Air Force, are carried out, first assuming isotropic and then anisotropic material properties. The correlation between test results and those predicted by isotropic theory is reasonably good. The anisotropic analysis gives predicted results which are in slightly more consistent agreement with the test data.

The procedure is also modified to give an isotropic deformation theory solution, which produces numerical results in a much shorter computer time than required for the incremental theory solution. In the case of the shear lag structure investigated, the results by the two theories are in very close agreement.

Creep test results of an 1100-F aluminum shear lag structure are also available. An analysis of this structure by the proposed incremental method is carried out and its predictions too are in reasonably good agreement with the test data. The 1100-F material is very nearly isotropic and no testing of structures exhibiting anisotropic creep is known to have been performed. Hence the anisotropic creep capability of the proposed method cannot be checked out against tests at this time. A sample calculation is nevertheless carried out for a hypothetical material having this characteristic.

The approach presented, which is simple in concept and execution, is found to be a reasonably good phenomenological model of an exceedingly complex physical problem. The accompanying digital computer program is believed to be very versatile, and well suited for the inclusion of any other types of material nonlinearity that may be of interest.

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SYMBOLS

$[\ ]$	Rectangular matrix
$\{\ \}$	Column matrix
q_i	Member load, any load acting directly on a member
σ_u	u^{th} ordinary stress component (normal or shear) at a node point
σ_{3N-2}, σ_{3N-1}, σ_{3N}	The normal stress components and shear stress component at node N - another designation for σ_u
$\bar{\sigma}_N$	Effective stress at node N
$\bar{\sigma}_{*_N}$	Relaxed effective stress at node N
σ_o	Reference stress in Ramberg-Osgood Equation
Γ_{um}	u^{th} stress component in linear redundant structure due to the m^{th} unit applied load
Γ_{uv}	u^{th} stress component in linear redundant structure due to the v^{th} unit initial strain
Γ_{im}	i^{th} member load in the redundant structure due to m^{th} applied load
Γ_{ij}	i^{th} member load in the redundant structure caused by a unit initial strain at the j^{th} member load
P_m	m^{th} applied load
ϵ_v	v^{th} component of initial strain
$\epsilon(p)_v$	v^{th} component of plastic strain
$\epsilon(c)_v$	v^{th} component of creep strain
$\bar{\epsilon}(p)_N$	Effective plastic strain at node N
$\bar{\epsilon}(c)_N$	Effective creep strain at node N
$\mathcal{E}$	Total (elastic, plastic, and creep) strain component
$\mathcal{E}_{xy}$	Engineering shear strain
ξ	Error
E	Young's modulus

T	Temperature
t^*	Equivalent time at start of creep cycle calculation
Δt	Cycle duration (elapsed time)
α, β, γ	Material constants in creep strain equation
α_{ij}	Subscripted anisotropic parameters
Δ	Increment (prefix)
n, θ	Nonlinear parameters in Ramberg-Osgood equation
k	Cycle designation, superscript
N	Nodal index
u, v	Nodal stress component or strain component index, related to the nodal index as indicated in equation
Y_{ij}	Simple directional characteristic stress
σ_{xx}, σ_{yy}	Uniaxial stress in the x-x, y-y direction
$\epsilon_{xx}, \epsilon_{yy}$	Plastic strain in the x-x, y-y direction due to uniaxial x-x, y-y stress
σ_{xy}	Shear stress in x-y plane
ϵ_{xy}	Shear strain in x-y plane

SECTION I

INTRODUCTION

With the introduction of high-speed, large-capacity, digital computers, a number of investigators (References 1,4,7) have adapted essentially linear matrix analysis methods to the solution of redundant structures where nonlinear material plasticity and creep properties are considered. These methods have been either iterative or non-iterative step-by-step numerical procedures.

The present work is an effort to explore, revise, and extend the matrix method of analysis in order to apply it to a range of practical aerospace structural problems exhibiting inelastic isotropic or inelastic anisotropic material behavior. Thus, the intent is to concentrate upon methods which are able to predict inelastic strain distributions in irregular idealized structures in a biaxial stress state where materials exhibit strain hardening and creep properties representative of those employed in aerospace construction.

A discussion of complete load reversal, although desirable for plastic fatigue studies, is not included because the theoretical foundation for such a procedure is apparently not yet fully developed. The present formulation of elastic, plastic and creep loading, followed by elastic unloading, while restrictive, is nevertheless of practical interest.

Of the proposed analytical methods, one by Denke (Reference 1) has developed naturally from the matrix force method of analysis and consists of including nonlinear plastic and creep terms in the equations for the gaps at those cuts which are required to make the structure statically determinate. The redundant forces, required to close the gaps and make the structure continuous, are obtained by solving these nonlinear equations by a Newton-Raphson procedure.

A second method reported by Kobayashi and Weikel (Reference 2) has been developed from the direct stiffness method. Here, forces occurring as a consequence of the inelastic effects are included in the nodal force equations. The nodal displacements (or displacement rates, where creep is considered) obtained by solving these equations impose equilibrium at the nodes under the action of internal loads, surface tractions, and prescribed displacements.

When the inelastic effects are accounted for by a flow (or incremental) theory, the deformation is an accumulation of increments each governed by the prevailing stress. Thus, when either of the methods just described is used, one must obtain (for the governing equations) a series of solutions, with one solution corresponding to each load increment. This requires a considerable amount of computer time even for medium-sized analyses.

The approach recommended here, in addition to being conceptually simple, does not require repeated matrix inversions. It was developed from that proposed by R. Gallagher, J. Padlog and others at Bell Aerosystem (References 3,4). Input data, as in the case with these other nonlinear analyses, are generated by an elastic analysis; however, for this approach, either the matrix force method or direct stiffness method can be used. The problem, formulated in terms of standard influence coefficients for applied load and initial strain, is reduced from a nonlinear to a linear one by using those strains obtained at the previous load level to approximate the current inelastic strains.

Development of the anisotropic analysis is based on an extension of the proposed anisotropic theory of Hu, Reference 11. The constant-anisotropic-coefficient assumption of Hu is replaced by one in which the coefficients are allowed to vary with the level of stress. The formulation is then a simple modification of the isotropic procedure. It is also shown that anisotropic creep can be included in a manner similar to the isotropic creep.

SECTION II

INELASTIC MATRIX METHODS

A. Formulation

An inelastic structural analysis can be carried out in two steps. The first is the standard elastic solution where internal stresses and accompanying strains are related through Hooke's law. The second step, the modification of this elastic system to include inelastic strains, is analogous to a procedure for including superimposed thermal strains. The inelastic strains are defined as the differences between the total strains and the elastic strains and are generally functions of the final stresses, not those of the linear, elastic state.

The simple, pin-jointed, single-redundant truss, pictured in Figure (1), illustrates the basic notions more clearly. All bars are considered to be elastic, except the vertical diagonal which can become plastic and has a stress-strain relation represented by the curve shown in Figure (2). The applied load P is large enough to cause member 3 to become plastic.

A solution might be obtained by first simply ignoring plastic strain in member 3 and assuming all members elastic. The resulting stress would then be of magnitude σ. The actual stress for member 3, of lower magnitude due to plastic yielding, is designated $\sigma^{(k)}$ and the associated inelastic strain $\epsilon^{(k)}$ in Figure (2). These stresses and strains are related in the following equation.

$$\sigma^{(k)} = \sigma + \Gamma\epsilon^{(k)} \qquad (1)$$

where:

$\sigma^{(k)}$ is the actual final stress

σ is the elastic stress

$\epsilon^{(k)}$ the inelastic strain

Γ the redundant elastic stress for a unit value of initial strain

To be more specific Γ is equal to the redundant stress in member 3 corresponding to a unit initial strain in member 3. Note that Γ must be negative to cause a reduction of stress in the diagonal member.

An important feature to be observed about Equation 1 is that, since the inelastic strain $\epsilon^{(k)}$ is a nonlinear function of the final stress $\sigma^{(k)}$, this equation is really a nonlinear relation to be solved

for $\sigma^{(k)}$. This characteristic will always be present in the analyses to be discussed in this report.

Equation 1 can be generalized to provide stresses in all members of the structure and provide for inelastic strains in all members for a variety of applied loads. This basic influence coefficient equation* is as follows:

$$\{\sigma_u\} = [\Gamma_{um}]\{P_m\} + [\Gamma_{uv}]\{\epsilon_v\} \qquad (2)$$

The σ_u's are the ordinary stress components at the various node points of the structure. An element of $[\Gamma_{um}]$ gives the u^{th} stress in the linear redundant structure due to a unit m^{th} applied load, and $\{P_m\}$ represents the actual applied loads. Also, an element of $[\Gamma_{uv}]$ gives the u^{th} stress component in the linear redundant structure due to a unit initial strain at the v^{th} stress location in the unloaded, statically determinate structure. Finally, an element of $\{\epsilon_v\}$ represents the actual initial strain at the v^{th} stress location.

The problem is now reduced in essence to the determination of the inelastic strains to use as the initial strains $\{\epsilon_v\}$ in Equation 2. For a structure in a load-temperature-time environment, this task can be rather formidable, because $\{\epsilon_v\}$ is a function of local temperature and time, as well as the local stress history.

B. Example Problem

Continuing our study of the simple truss example of Figure (1), we now allow all members to go plastic. The exact results for the deformation and stresses in the truss, for nonlinear properties, are easily obtained by direct numerical solution of the equations, and hence will be used without development.

The step-by-step finite element method for determining the stresses $\{\sigma_u\}$ and strains $\{\epsilon_v\}$ involves the use of Equation 2 and a nonlinear stress-strain relation. This relation will be assumed to be a piecewise linear approximation of the Ramberg-Osgood stress-strain relation.

$$\epsilon = \frac{\sigma}{E} + \epsilon$$

*The derivation of matrices $[\Gamma_{um}]$ and $[\Gamma_{uv}]$ is given in Appendix VI.

where ϵ denotes the inelastic (or plastic) strains and is given by

$$\epsilon = \frac{3}{7}\left(\frac{\sigma}{\sigma_o}\right)^{\theta-1}$$

and where

$\mathcal{E}$ = total strain

σ = member stress

E = Young's modulus

σ_o = reference stress (stress at secant modulus of 0.7E)

θ = nonlinear parameter

The first step, in applying the finite element method, is to obtain the influence coefficient matrices $[\Gamma_{um}]$ and $[\Gamma_{uv}]$ for the linear, redundant structure. This requires specification of the geometry of the structure and the (linear) material properties of the individual structural elements. The geometry of the example truss problem is given in Figure (1). The material is assumed to be an aluminum alloy with the following constants.

$$E = 10^7 \text{ psi}, \quad \sigma_o = 10^5 \text{ psi}, \quad \theta = 10$$

The approximate stress-strain curve used matches the Ramberg-Osgood values at 2000 psi stress intervals.

In this case, we find

$$[\Gamma_{um}] = \begin{bmatrix} .207 \\ -.293 \\ .707 \end{bmatrix}, \quad [\Gamma_{uv}] = 10^7 \times \begin{bmatrix} -.414 & .207 & .207 \\ .586 & -.293 & -.293 \\ .586 & -.293 & -.293 \end{bmatrix}$$

for the case of the single applied load P.

C. Step-by-Step Methods

References 3 and 4 present what appears to be the simplest possible approach to this problem from a computational standpoint.* A non iterative step-by-step calculation is performed in which all quantities including the initial strains ϵ_v are incremented and then assumed to remain constant in the ensuing load interval. The inherent difficulty in this approach is to establish the connection between successive steps. Two methods, both of which involve the initial strains from the prior step to predict quantities in the current step are suggested. It is anticipated that by controlling the size of the interval one may achieve any degree of accuracy.

The development herein is discussed only in the detail necessary to analyze the redundant truss of Figure (1) being loaded for the first time into the plastic range. Generalization to biaxial plasticity and creep phenomena are discussed in the succeeding sections.

The step-by-step procedure for solving the problem is introduced by rewriting Equation 2 in the form

$$\left\{\sigma_u^{(k)}\right\} = [\Gamma_{um}]\left\{P_m^{(k)}\right\} + [\Gamma_{uv}]\left\{\epsilon_v^{(k-1)}\right\} \qquad (3)$$

where

k is the cycle designation

This can be regarded as the fundamental equation for the non-iterative, step-by-step methods. The idea in formulating this equation, as indicated by the cycle designating superscript is that the initial strains of the previous cycle can be used to approximate the initial strains of the current cycle. The strains of the previous cycle may be incorporated in several ways, two of which constitute the constant stress and the constant strain methods of analysis.

D. Constant Stress Method

As indicated previously, in the step-by-step procedure considered here, one enters the k^{th} cycle with applied loads $\left\{P_m^{(k)}\right\}$ and initial strains $\left\{\epsilon_v^{(k-1)}\right\}$, the latter evaluated during the preceding cycle.

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*These methods make use of devices previously used by others to solve inelastic problems; for example, S. S. Manson at the Lewis Research Laboratory, NASA, Cleveland, Ohio, has previously carried out inelastic analyses of turbine discs involving somewhat similar techniques (Reference 5).

The first operation of the current cycle is to determine $\left\{\sigma_u^{(k)}\right\}$ from Equation 3 by direct substitution. The second operation is a determination of $\left\{\epsilon_v^{(k)}\right\}$ for use in the next cycle. The constant stress method does this in the most obvious way, by reading from the given stress-strain curve the plastic strains $\left\{\epsilon_v^{(k)}\right\}$ corresponding to the $\sigma_u^{(k)}$'s (the reason for the name "constant stress" is thus apparent). The operation is indicated schematically in Figure (2).

The results of the application of this method to the example truss problem are shown in Figure (4), where the stress in the vertical member (Bar #3) has been plotted versus load. These results display a striking defect of the method due to the development of a sudden and catastrophic divergence, whose onset depends upon step size. This dependence is such that any attempt to improve accuracy by reducing step size only hastens the occurrence of divergence. An explanation of this behavior is given in Appendix III. Because of this defect, the constant stress method in this form must be eliminated from consideration as an acceptable method for general use.

E. Constant Strain Method

The first operation of the constant strain method is exactly the same as the first operation of the constant stress method; $\sigma_u^{(k)}$ is evaluated by direct substitution in Equation 3. Thereafter, one determines $\epsilon_v^{(k)}$ for use in the next cycle as follows. Referring to Figure (3), for each member, point A is determined with stress-strain coordinates $\sigma_u^{(k)}$ and $\sigma_u^{(k)}/E + \epsilon_v^{(k-1)}$. A relaxed stress $\sigma_u^{*(k)}$ is now calculated with the same total strain, corresponding to point B on the given stress-strain curve. Note that here the total strain, rather than stress, remains unchanged--hence the name "constant strain" method. The required initial strain $\epsilon_v^{(k)}$ is the inelastic strain $\epsilon^{(k)}$ corresponding to the relaxed stress, as indicated on Figure (3).

The results of applying the constant strain method to the truss problem, for the three step sizes 5000, 500 and 50 lb., are shown in Figure (5). The accuracy, for a given step size, is not as good as that of the constant stress method, but the analysis is now free of any instability. The constant strain method is therefore selected for further use herein. The discussion of the step size and of a method of monitoring it is left for a later section.

SECTION III

ISOTROPIC ELASTIC-PLASTIC ANALYSIS

A. Biaxial Theory

Having presented the simple truss example of the step-by-step procedure, we proceed now to the case of more practical interest -- a biaxially stressed structure. The new procedure is identical to the one already discussed for the simple truss with one exception. Because of the biaxial stress we can no longer work directly from the stress-strain curve to obtain the plastic strains for use in Equation 3; instead, we must employ the well-known concept of an "effective" stress-strain relationship in conjunction with a von Mises type yield condition and the associated incremental flow relations.

The biaxial theory is described by a summary of the steps to be used as a guide for a detailed description which follows. The constant strain method used here is a step-by-step procedure which, after incrementing the applied load, can be applied in four parts:

1. Obtain the stress components at each node using the basic Equation 3 by assuming the initial strains from the previous load level.

2. Using these stresses, calculate an effective stress at each node.

3. Assume that the effective stress-strain relation for the material, modified by including the elastic strain, corresponds to data measured in a simple uniaxial tension test. Using this, calculate the effective strain corresponding to the effective stress.

4. Using the incremental flow relations, determine the inelastic strain increments. The proportionality constant in these equations is the ratio of the effective strain increment to the effective stress.

At this point in the calculation, the applied load can be incremented again and the cycle repeated.

When calculating the ordinary stresses $\left\{\sigma_u^{(k)}\right\}$ for the k^{th} load level using Equation 3 (Step 1), it is convenient to re-identify these stresses by means of a new subscript N, as follows:

$$\begin{Bmatrix} \vdots \\ \sigma_u^{(k)} \\ \vdots \end{Bmatrix} = \begin{Bmatrix} \vdots \\ \sigma_{3N-2}^{(k)} \\ \sigma_{3N-1}^{(k)} \\ \sigma_{3N}^{(k)} \\ \vdots \end{Bmatrix} \qquad (4)$$

The stresses are thus arranged in groups of three components (two normal and one shear) at each node point N.

We now calculate the corresponding effective stresses $\bar{\sigma}_N^{(k)}$ for each of the nodes from the von Mises type formula (Step 2)

$$\bar{\sigma}_N^{(k)} = \left[\left(\sigma_{3N-2}^{(k)}\right)^2 - \left(\sigma_{3N-2}^{(k)}\right)\left(\sigma_{3N-1}^{(k)}\right) + \left(\sigma_{3N-1}^{(k)}\right)^2 + 3\left(\sigma_{3N}^{(k)}\right)^2\right]^{\frac{1}{2}} \qquad (5)$$

Note that by this definition $\bar{\sigma}_N^{(k)}$ must be positive and is proportional to the octahedral shear stress. This formula together with the stress-strain data constitutes the strain hardening criterion.

We now go to the tensile stress-strain curve (which is also the $\bar{\sigma}$ vs $\bar{\sigma}/E + \bar{\epsilon}(p)$ curve) for the material of interest and, using the constant strain method, read from it the corresponding effective plastic strain $\bar{\epsilon}(p)_N^{(k)}$. This operation (Step 3) is identical to that previously described for the uniaxial case on page 7.

In accordance with the flow theory of plasticity, the increment in the effective strain $\Delta\bar{\epsilon}(p)_N^{(k)}$ over that of the preceding interval must be calculated (Step 4). The increment will be either positive or zero, depending upon whether plastic loading or elastic unloading (or reloading) is taking place. Thus,

$$\Delta\bar{\epsilon}(p)_N^{(k)} = \bar{\epsilon}(p)_N^{(k)} - \bar{\epsilon}(p)_N^{(k-1)} \qquad (6a)$$

when $\bar{\sigma}_N^{(k)}$ is greater than any previous $\bar{\sigma}_N$ (inelastic strain increasing)

$$\Delta\bar{\varepsilon}(p)_N^{(k)} = 0 \qquad (6b)$$

when $\bar{\sigma}_N^{(k)}$ is smaller than a previous $\bar{\sigma}_N$ (elastic unloading or reloading, inelastic strain constant)

The increments in the ordinary plastic strain components may now be obtained using a Prandtl-Reuss incremental relationship.

$$\Delta\varepsilon(p)_{3N-2}^{(k)} = \frac{\Delta\bar{\varepsilon}(p)_N^{(k)}}{\bar{\sigma}_N^{(k)}} \left[\sigma_{3N-2}^{(k)} - 1/2\ \sigma_{3N-1}^{(k)}\right]$$

$$\Delta\varepsilon(p)_{3N-1}^{(k)} = \frac{\Delta\bar{\varepsilon}(p)_N^{(k)}}{\bar{\sigma}_N^{(k)}} \left[\sigma_{3N-1}^{(k)} - 1/2\ \sigma_{3N-2}^{(k)}\right] \qquad (7)$$

$$\Delta\varepsilon(p)_{3N}^{(k)} = \frac{\Delta\bar{\varepsilon}(p)_N^{(k)}}{\bar{\sigma}_N^{(k)}} \left[3\sigma_{3N}^{(k)}\right]$$

The total, ordinary plastic strain components are obtained by addition,

$$\begin{Bmatrix} \vdots \\ \varepsilon(p)_{3N-2}^{(k)} \\ \varepsilon(p)_{3N-1}^{(k)} \\ \varepsilon(p)_{3N}^{(k)} \\ \vdots \end{Bmatrix} = \begin{Bmatrix} \vdots \\ \varepsilon(p)_{3N-2}^{(k-1)} \\ \varepsilon(p)_{3N-1}^{(k-1)} \\ \varepsilon(p)_{3N}^{(k-1)} \\ \vdots \end{Bmatrix} + \begin{Bmatrix} \vdots \\ \Delta\varepsilon(p)_{3N-2}^{(k)} \\ \Delta\varepsilon(p)_{3N-1}^{(k)} \\ \Delta\varepsilon(p)_{3N}^{(k)} \\ \vdots \end{Bmatrix} \qquad (8)$$

These components together with the new applied loads $P_m^{(k+1)}$ may be substituted in Equation 3 to obtain $\sigma_u^{(k+1)}$ in the next load cycle.

B. Determination of Calculation Step Size

It should be noted that, according to the constant strain method, every predicted value of effective strain $\bar{\epsilon}(p)_N^{(k)}$ together with its accompanying value of effective stress $\bar{\sigma}_N^{(k)}$ constitutes an approximation to a point on the actual effective stress-strain curve. The excellence of the approximation is directly related to the loading increment, as is shown in the truss results Figure (5). Thus it is only necessary to monitor this agreement for one or more of the critically loaded nodes to determine whether the step size is satisfactory. This is illustrated below in connection with the shear lag structure investigation.

C. Description of Shear Lag Structure

Several very useful tests have been performed for the Air Force upon shear lag structures (Reference 7). The structure, loaded as shown in Figure (6), is an integrally machined part of 2024-T4 aluminum alloy stiffened along the loading (y) axis. The stiffener is tapered in thickness from each end towards the center of the structure.

This structure was chosen originally because it is simple to work with and well adapted to analysis by both matrix methods when appropriate idealizations are employed. When tension forces are applied to the ends of the stiffener, high stress gradients are induced in a manner analogous to those encountered in aircraft structures.

The material properties essential to this analysis were obtained from tension tests reported in Reference 7. These tests were performed on coupons, machined from the parent plate, in the longitudinal or x-direction and the transverse or y-direction of Figure (6). The data resulting from these tests, Figure (11), indicate the presence of a considerable degree of anisotropy. In the present study, three piecewise linear representations of stress-strain curves were fitted to these points; two, R01 and R02, in Figure (11), are equivalent to Ramberg-Osgood curves used in Reference 7; the third, R02M, is a Grumman modification. The modulus of elasticity of all the curves is taken as 10.3×10^6 psi. Note also that the maximum strains recorded are of the order of 0.010 in/in, whereas the maximum strains reached in the shear lag tests are around 0.020 in/in. Thus there is some doubt as to whether the idealized curve correctly represents the test material in this high strain region.

The locations of the strain gages for the test of the stiffened plate are shown in Figure (13). The plate was loaded by applying tension to the stiffener in steps of 1000 pounds to 6000 pounds, gage readings being taken at each step. It was then unloaded in steps of 1500 pounds to zero, and finally progressively loaded to failure. Buckling occurred at a load of 23,000 pounds and fracture at 25,800 pounds. Data from this test are plotted on Figures (14) through (22).

D. Elastic Shear Lag Structure Analysis

The idealizations of the upper right quandrant of the shear lag structure for a direct stiffness and a force method analysis are shown in Figures (7) and (8) respectively. A typical element of the force method analysis, Figure (9), consists of conventional bars and rectangular shear panels. Many previous idealizations have omitted the Poisson's Ratio effect. The present idealization, however, incorporates this effect in the manner described in Appendix II.

The basic element of the stiffness approach consists of a cluster of four "Turner triangles" (References 8,9) to form a rectangle as shown in Figure (10). The manner of obtaining the stresses is discussed in Appendix I.

An elastic analysis under a unit applied load was performed by both the force and the stiffness methods. These results are compared in Appendix I.

The inelastic analysis can be made using either of the two approaches (stiffness or force method). For the present investigation, only the force method is used.

E. Flow Theory Shear Lag Structure Analysis

The inelastic analysis was carried out using the Fortran 2 program listed in Appendix V. This program is capable of carrying out isotropic or anisotropic, plastic or creep flow theory analyses. The flow charts and instructions for preparation and submission of data are also included in Appendix V.

Before comparing the analytical and test results, let us look at a plot of the tensile stress-strain curve, R02M, and compare it with the predicted effective stress-strain relationship for various step sizes. Such a comparison is found in Figure (12) for the node corresponding to the center of the specimen which is the point of highest strain in the structure. It can be seen there that for a step size of $\Delta P = 500$ lb., the agreement is rather poor. For $\Delta P = 50$ lb. the agreement is much better, while for $\Delta P = 5$ lb., the predicted value lies directly on the stress-strain curve. The IBM 7094 computer time for this best result and a maximum load of P = 16,760 lb. is approximately 20 minutes.

The predicted strain distributions are shown in Figures (14) through (21), together with the corresponding test values, along the two strain gage lines. In these plots, the calculated results are linearly interpolated values between node points. Figures (14) and (15) give elastic results; the agreement with test data is seen to be rather good, giving the necessary confidence in the accuracy of the basic influence coefficient matrices. It is observed also that the specimen achieves its basic purpose of displaying a pronounced shear lag effect with the highest strain occurring at the central node, as expected.

As the applied load increases through 7070 lbs., the strains at the central node become plastic. During the tests, the strain gages continued to function through an applied load level of 14,600 lb.; beyond this point the x-gage failed to record. The y-gage failed also above a load level of 16,760 lb., which, consequently, is the highest level considered in the comparison of test and analysis even though the plate did not buckle until P = 23,000 lb.

The proportional limit for the tes specimen material occurs at a strain of approximately .004 in/in, as shown in Figure (11). Bearing this in mind during an examination of Figures (16) through (21), it is seen that plastic behavior is primarily confined to a fairly small region around the central node, and we thus have a case of contained plasticity. The analysis predicts a very pronounced strain redistribution extending somewhat beyond this region. This is indicated by a comparison of the plastic results with extrapolated elastic results shown as dotted curves on Figures (20) and (21).

As for agreement with test values, the analysis substantially underpredicts the strain gage readings where plasticity is most pronounced. Since the elastic results agree so much better, one must assume that the difficulty lies somewhere in the plasticity part of the correlation. It was mentioned previously that the idealization of the stress-strain curve R02M of Figure (11) was open to question at the high strain end because of the absence of test points. Accordingly, an additional run was made for a revised curve extending horizontally beyond the last indicated test point. The plastic strains at the critical central node are increased approximately 10% by so doing. This represents an appreciable closing of the gap, but the gap nevertheless remains.

Calculations were also made based upon the previously mentioned stress-strain curves R01 and R02 of Figure (11). The results for the central node are shown on Figure (22). As might be expected, they depart appreciably from the R02M predictions.

F. Deformation Theory Analysis

Solutions by a deformation theory have traditionally been considered to be more easily obtainable than flow theory solutions. This is, of course, because only the stresses at the final applied load level need be considered, rather than the stress histories developed during loading. It is therefore of interest to determine whether similar benefits are attainable in the case of the finite element analyses currently being considered.

Once again, a solution of Equation 3 is required, this time such that the initial strains ϵ_v satisfy the deformation theory of plasticity. This can be accomplished as follows. Equation 3, the "k^{th} cycle" stress equation of the preceding section, can be used intact if it is understood that $P_m^{(k)}$ is the peak load at which the results are required and does not change from one cycle to another as before. We must iterate to a solution in order to obtain a satisfactory approximation to the plastic strains.

The intra-cycle procedure employed for the determination of the equivalent strain for the k^{th} cycle is the same as before, namely the constant strain method. At this point, however, the equivalent strain itself, not its increment, is resolved into the node plastic strains by utilizing an engineering adaptation of the incremental relations, Equations 7, thus:

$$\epsilon(p)^{(k)}_{3N-2} = \frac{\bar{\epsilon}(p)^{(k)}_N}{\bar{\sigma}^{(k)}_N} \left[\sigma^{(k)}_{3N-2} - 1/2\ \sigma^{(k)}_{3N-1}\right]$$

$$\epsilon(p)^{(k)}_{3N-1} = \frac{\bar{\epsilon}(p)^{(k)}_N}{\bar{\sigma}^{(k)}_N} \left[\sigma^{(k)}_{3N-1} - 1/2\ \sigma^{(k)}_{3N-2}\right] \qquad (9)$$

$$\epsilon(p)^{(k)}_{3N} = \frac{\bar{\epsilon}(p)^{(k)}_N}{\bar{\sigma}^{(k)}_N} \left[3\sigma^{(k)}_{3N}\right]$$

These are now available for the stress equation of the next cycle.

Three analyses, one at P = 11,600 lbs., one at P = 14,600 lbs., and one at P = 16,760 lbs. were performed on the shear lag specimen by this deformation theory procedure. The results were practically identical with those shown in Figures (16) to (21). The convergence to each of these results was obtained after less than thirty cycles of iteration. The machine time for each calculation was approximately four minutes.

In the case of solutions like this, where the two analyses give practically identical results, the deformation approach is naturally very attractive because of the greatly reduced machine time. However, the question remains of determining when to expect the results to agree in this manner.

The IBM program presented in Appendix V cannot be used for deformation theory analyses. However, minor changes can be made in the program to permit calculations of this type to be carried out.

SECTION IV

ISOTROPIC ELASTIC-PLASTIC-CREEP ANALYSIS

A. Introduction to Creep Theory

Strains due to creep constitute an additional form of inelastic strain, and can be handled in a way analogous to that already discussed for biaxial plasticity by the flow theory. It is only necessary to select a method for evaluating these time-dependent strains based upon the material properties and to add them to the plastic strains prior to insertion in the basic Equation 3.

A familiar relationship used to match the creep behavior in a tensile creep test, performed at constant stress and constant temperature, is (Reference 4):

$$\epsilon(c) = \alpha t^{\gamma}(e^{\beta\sigma}-1) \qquad (10)$$

in which

- $\epsilon(c)$ is the tensile creep strain
- t is the elapsed time
- σ is the constant tensile stress
- α,β,γ are empirical constants for the particular test temperature

For this analysis the assumption is made that there exists an effective creep strain $\bar{\epsilon}(c)$ in a biaxial situation which can be calculated using Equation 10. In doing this the stress σ is taken to be the von Mises effective stress obtained from Equation 5. The further assumption is made that this effective creep strain can be resolved into nodal creep strains by use of a Prandtl-Reuss type of flow law.

The creep strain calculation must be generalized to situations in which the stresses vary with time. One well-known procedure for doing this, the strain-hardening rule (Reference 4), has been determined to be most appropriate for the present purposes. Its use will be described presently in connection with the k^{th} calculation cycle.

B. Creep Theory Details

The calculation cycle follows a sequence similar to that described previously for the isotropic elastic-plastic analysis. An additional step is necessary, just after the plastic strains are obtained, to determine the creep strains. The intra-cycle order of calculation is as follows.

Entering the k^{th} cycle with applied loads $\left\{P_m^{(k)}\right\}$, and with the initial strains, plastic and creep, as calculated during the preceding cycle, the stresses $\left\{\sigma_u^{(k)}\right\}$ are calculated from Equation 3.

The effective stresses $\left\{\bar{\sigma}_N^{(k)}\right\}$ obtained from Equation 5 are used to obtain the effective plastic strains.

The creep strain increments at each node, for a specified time step, are now determined by the strain hardening rule which relates the strain at a node to the corresponding stress and strain for the previous cycle by the introduction of an assumed elapsed time.

Referring to Figure (23), one goes to the constant effective stress-temperature curve $(\bar{\sigma}^k, T^k)$ relevant to the node and the cycle, and locates upon it the point with ordinate $\bar{\epsilon}(c)^{(k-1)}$. The corresponding abscissa, designated $t^{*(k)}$, is called the reference time and is generally different from the actual elapsed time at the start of the cycle. The required effective creep strain increment $\Delta\bar{\epsilon}(c)_N^{(k)}$ is that corresponding to the increase in time from t^{*k} to $(t^{*k} + \Delta t^k)$ as shown on Figure (23), Δt^k being the selected calculation time increment.

The increment $\Delta\bar{\epsilon}(c)^k$ is substituted into the Prandtl-Reuss type incremental relations, Equations 7, together with the stresses indicated there. The creep initial strains are then obtained as in Equation 8.

In summary, the steps in the k^{th} calculation take the following order:

(1) Evaluation of Equation 3 to obtaining the stress components, $\sigma_u^{(k)}$

(2) The calculation of effective stress according to Equation 5

(3) The determination of the node plastic strains

(4) The determination of the node creep strains

(5) The addition of the nodal plastic and creep strains to give the initial strains for the next cycle.

C. Description of Structure and Tests

The description of the shear lag structure to be analyzed in this section and tests for the material properties may be found in Reference 7. The shear lag structure was manufactured from 1100-F aluminum. It was of the same physical dimensions as the structure of Figure (6). The idealization of the upper right quadrant remains unchanged.

Material properties for the creep analysis were obtained from uniaxial strain-time tests for constant tensile stress. The temperature at which these tests were conducted was 206°C, the temperature identical to that of the structural test. The curves for these tests are presented in Figure (24) together with the fitted curves from Equation 10. The constants of the equation were obtained from Reference 7 and are as follows:

$$\alpha = 0.650 \times 10^{-4}$$

$$\gamma = 0.500$$

$$\beta = 0.700 \times 10^{-3} \text{ in}^2/\text{lb}$$

Ordinary tensile stress-strain tests were performed at room temperature on coupons cut from the x and y orientations of the plate material. The data and faired curve are presented in Figure (25). Tensile stress-strain data for 206°C are also plotted here. These data were obtained from the intersections of the test curves on the zero time axis in Figure (24). A piecewise linear representation TC1 was fitted to this latter data.

The location of the strain gages on the structure is given in Figure (26). The shear lag specimen was tested for a total of three hours at 206°C. An initial applied load of 1600 lb. was increased to 2020 lb. at the end of the first hour. It was held constant thereafter to the end of the test.

D. Results of Creep Shear Lag Analysis

The predicted strain distributions along the x-axis and along the section x = 1 in. at t = 0.06 hr., t = 1.10 hr., and t = 3.00 hr. elapsed times, are given in Figures (27) through (32), together with the experimental data of Reference 7. Test data are not available for the y-node strains at the center node, and so this correlation point of critical significance does not exist.

The curve of Figure (33), effective stress versus strain at the central node, exhibits the shapes characteristic to the various regions of the load-time sequence. The initial linear segment, representing elastic loading is followed by the region of the negative curvature representing loading into the plastic range, all at assumed zero time. Thereafter, the applied load remains constant for one hour, during which time there is a stress redistribution in the structure due to creep. This particular node unloads, as evidenced by the reduction in effective stress, although the total strain is growing continuously. The applied load is now increased to 2020 lb. Because of the previous elastic strain recovery, the effective stress at first goes up elastically, and then becomes plastic once more. Once the applied load reaches its final value, redistribution due to creep effects again takes place.

In the initial stages of creep the curve is very sensitive to time increment size and it is necessary to choose exceptionally small time

increments in this region if good accuracy is to be achieved. The time increments employed are as shown in Figure (33).

Considering the simplicity of the expressions employed to describe as complex a phenomenon as creep and the liberal assumptions made in the process, the correlation between analysis and experiment, as evidenced by the preceding graphs and also by Figures (34) and (35), is surprisingly good.

SECTION V

ANISOTROPIC ELASTIC-PLASTIC ANALYSIS

A. Anisotropy in Structures

Several "expanding yield surface" theories for extending the isotropic plastic theory to provide for anisotropy have become available within the last two decades. Each is based on experimentally determined parameters and, therefore, each is biased in favor of specific test data. The complexity and amount of testing required to obtain these parameters differ considerably. Under these circumstances, no single theory can be completely acceptable, but it is thought that a suitable theory must, at least, be capable of evaluating the type of anisotropy associated with biaxially stressed structures used in flight vehicle design without being unduly complex in application. It would be desirable to have the procedure based on a well-known, accepted theory.

A particular type of anisotropy, the so-called "orthotropic symmetric" type, develops during a cold rolling process where the material is lengthened and thinned with no appreciable change in width. Since cold rolled sheet and plate are frequently used in aerospace structures, this type of anisotropy may be anticipated and is considered here.

A theory proposed by Hill, Reference 10, has been widely accepted as the most straightforward extension of the isotropic theory. The formulation, however, is not very convenient for numerical step-by-step computation. A modification of Hill's theory proposed by Hu, Reference 11, however, is very tractable to formulation into the matrix inelastic program discussed previously in this report. The Hu procedure has two distinct advantages:

(1) It employs a von Mises type hardening surface, associated flow law and effective stress-strain relationships in appropriate form.

(2) It requires a minimum of material data: simple uniaxial and shear stress-strain tests on coupons cut in the directions of the orthotropy.

B. Hu's Strain Hardening Theory

A summary of Hu's theory is presented to establish its limitations and provide background for the necessary modifications to obtain a more general theory.

The isotropic expressions for effective stress, Equation 5, and associated incremental relations, Equations 7, are modified by means of anisotropic parameters (α_{ij}). These are constants in Hu's theory. Here it is more convenient to introduce the 1-1, 2-2, etc., directions instead

of x-x, y-y, etc. The modified expression for the effective stress is

$$\bar{\sigma} = \left[\alpha_{12}(\sigma_{11} - \sigma_{22})^2 + \alpha_{23}(\sigma_{22} - \sigma_{33})^2 + \alpha_{31}(\sigma_{33} - \sigma_{11})^2 + 3\alpha_{44}\sigma_{12}^2 + 3\alpha_{55}\sigma_{23}^2 + 3\alpha_{66}\sigma_{31}^2\right]^{\frac{1}{2}} \tag{11}$$

The incremental flow equations are

$$\begin{aligned}
d\epsilon_{11} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[\alpha_{11}\sigma_{11} - \alpha_{12}\sigma_{22} - \alpha_{31}\sigma_{33}\right] \\
d\epsilon_{22} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[-\alpha_{12}\sigma_{11} + \alpha_{22}\sigma_{22} - \alpha_{23}\sigma_{33}\right] \\
d\epsilon_{33} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[-\alpha_{31}\sigma_{11} - \alpha_{23}\sigma_{22} + \alpha_{33}\sigma_{33}\right] \\
d\epsilon_{12} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[3\alpha_{44}\sigma_{12}\right] \\
d\epsilon_{23} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[3\alpha_{55}\sigma_{23}\right] \\
d\epsilon_{31} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[3\alpha_{66}\sigma_{31}\right]
\end{aligned} \tag{12}$$

Equations 11 and 12 are written for the case where the reference axes are the principal axes of anisotropy.

The anisotropic parameters are determined by means of a total of six, simple, directional, stress-strain tests (i.e., uniaxial and shear tests), where, alternately, all stress components are equal to zero except one. From each of the six tests a characteristic stress, such as an approximate yield stress, is read off. Then substituting each of these results into Equation 11 in succession, we may write

$$\begin{aligned}
\alpha_{11} &= \alpha_{12} + \alpha_{31} = \left(\frac{K}{Y_{11}}\right)^2 & \alpha_{44} &= \frac{1}{3}\left(\frac{K}{Y_{12}}\right)^2 \\
\alpha_{22} &= \alpha_{23} + \alpha_{12} = \left(\frac{K}{Y_{22}}\right)^2 & \alpha_{55} &= \frac{1}{3}\left(\frac{K}{Y_{23}}\right)^2 \\
\alpha_{33} &= \alpha_{31} + \alpha_{23} = \left(\frac{K}{Y_{33}}\right)^2 & \alpha_{66} &= \frac{1}{3}\left(\frac{K}{Y_{31}}\right)^2
\end{aligned} \tag{13}$$

where K = the effective characteristic stress

Y_{ij} = the simple directional characteristic stress

It remains to assume an effective stress-effective strain relationship. Hu shows that it is acceptable to assume the stress-strain data associated with one particular simple tension test as the effective relationship. It can be seen from Equations 13 that this implies setting one α_{ii} equal to unity.

Thus, in particular for $\alpha_{22} = 1$, from Equations 11 and 12:

$$\begin{aligned} \bar{\sigma} &= \sigma_{22} \\ d\bar{\epsilon} &= d\epsilon_{22} \end{aligned} \tag{14}$$

Now consider Equation 12 for simple stress-strain tests in the other directions. Then

$$\begin{aligned} d\epsilon_{11} &= \alpha_{11}\sigma_{11}\frac{d\bar{\epsilon}}{\bar{\sigma}} \\ d\epsilon_{33} &= \alpha_{33}\sigma_{33}\frac{d\bar{\epsilon}}{\bar{\sigma}} \\ d\epsilon_{12} &= 3\alpha_{44}\sigma_{12}\frac{d\bar{\epsilon}}{\bar{\sigma}} \\ d\epsilon_{23} &= 3\alpha_{55}\sigma_{23}\frac{d\bar{\epsilon}}{\bar{\sigma}} \\ d\epsilon_{31} &= 3\alpha_{66}\sigma_{31}\frac{d\bar{\epsilon}}{\bar{\sigma}} \end{aligned} \tag{15}$$

or

$$\frac{d\bar{\epsilon}}{\bar{\sigma}} = \frac{1}{\alpha_{11}}\frac{d\epsilon_{11}}{\sigma_{11}} = \frac{1}{\alpha_{33}}\frac{d\epsilon_{33}}{\sigma_{33}} = \frac{1}{3\alpha_{44}}\frac{d\epsilon_{12}}{\sigma_{12}} = \frac{1}{3\alpha_{55}}\frac{d\epsilon_{23}}{\sigma_{23}} = \frac{1}{3\alpha_{66}}\frac{d\epsilon_{31}}{\sigma_{31}}$$

These equations say that, with the anisotropic parameters constant for strain hardening, the simple, stress-incremental strain relations must be proportional to the effective stress-incremental strain relationship. The implication is that the integrated forms of Equations 15, that is, the simple, directional stress strain curves, are thus prescribed. These may or may not be a reasonable fit to the test data for the material of interest. Obviously, only when the fit is good can one hope for Hu's theory to give acceptable results for all types of loading.

Based upon the Hu theory, it becomes relatively easy to obtain anisotropic solutions using the previously developed isotropic inelastic procedure and corresponding digital computer program. It is only necessary to substitute the appropriate anisotropic constants for their

isotropic counterparts, for which $\alpha_{11} \ldots \alpha_{66} = 1$, $\alpha_{12} = \alpha_{23} = \alpha_{31} = \frac{1}{2}$.

C. Extension of Hu's Theory

It is in the determination of the parameters that an extension to Hu's theory is made. Except for the special case pointed out in the preceding section, these parameters should not be constant for a strain hardening material, but should be variables dependent upon stress level. The objective, obviously, is to determine the variation in a manner that allows for all of the simple, directional stress-strain curves to be correctly reproduced. This can be accomplished by a consideration of plastic work.

In the current approach, one continues with the assumption of the existence of an effective stress-strain relationship. Then the basic notion is that the anisotropic parameters are determined such that, for equal amounts of plastic work done during simple directional stress strain tests in all directions, the effective stress level reached will be identical.

Accordingly, for a tensile specimen in the 1-1 direction, one calculates the plastic work w performed during a uniaxial test by the formula

$$w = \int \sigma_{11} \, d\epsilon_{11} = \int \bar{\sigma} \, d\bar{\epsilon} \qquad (16)$$

Let the corresponding maximum stress reached be identified by the superscript (I), i.e. σ_{11}^{I}. For a similar test in the 2-2 direction, and for which the amount of plastic work performed is identical, the corresponding maximum stress reached is σ_{22}^{I}. Since the amounts of work done in the two cases are the same, σ_{11}^{I} and σ_{22}^{I} correspond to the same effective stress $\bar{\sigma}^{I}$. By means of Equation 11 we have

$$\left(\bar{\sigma}^{I}\right)^2 = \alpha_{11}\left(\sigma_{11}^{I}\right)^2 = \alpha_{22}\left(\sigma_{22}^{I}\right)^2 \qquad (17)$$

This expression constitutes a relationship defining α_{11} and α_{22} as functions of $\bar{\sigma}$. Similar relationships can clearly be found for the other α_{ii}'s. Thereafter, the α_{ij}'s can be determined as functions of $\bar{\sigma}$ by recourse to the α definitions of Equations 13.

It is convenient to again select the 2-2 direction stress-strain curve as the one defining the effective stress-strain relationship. This results in α_{22} being equal to unity once more.

The actual evaluation of the other α's as functions of $\bar{\sigma}$ follows easily. Figure (36) indicates schematically how the plastic work done in each simple directional stress-strain test can be plotted as a function of stress. Then for a given amount of plastic work, and reading off the corresponding stresses σ_{11}^{I} and σ_{22}^{I} for example, by Equation 17 one finds that

$$\alpha_{11} = \frac{\left(\sigma_{22}^{I}\right)^2}{\left(\sigma_{11}^{I}\right)^2} \tag{18}$$

Using this and similar relationships in the other directions, curves representing all of the α's as functions of $\bar{\sigma}$ may be constructed. The incorporation of this information in the step-by-step calculation procedure is discussed in the next section.

D. Anisotropic Theory Details

The detailed step-by-step calculation procedure to be followed in the case of anisotropic material in a biaxially stressed structure is very similar to that previously discussed for isotropic materials. Accordingly, only the differences will be stressed.

As before, one starts the k^{th} calculation cycle by evaluating $\{\sigma_u^{(k)}\}$ by means of Equation 3. This operation employs the initial strains of the preceding cycle $\{\epsilon_v(p)^{(k-1)}\}$.

The next operation is to evaluate the effective stresses at each of the nodes, $\bar{\sigma}_N^{(k)}$, by Equation 11, modified for the biaxial case to

$$\bar{\sigma}_N^{(k)} = \left[\alpha_{11}^{(k-1)}\left(\sigma_{3N-2}^{(k)}\right)^2 - 2\alpha_{12}^{(k-1)}\left(\sigma_{3N-2}^{(k)}\right)\left(\sigma_{3N-1}^{(k)}\right) + \left(\sigma_{3N-1}^{(k)}\right)^2 + 3\alpha_{44}^{(k-1)}\left(\sigma_{3N}^{(k)}\right)^2\right]^{\frac{1}{2}} \tag{19}$$

We continue here our assumption that the 2-2 direction has been selected as that in which the effective stress-strain relationship is defined; hence $\alpha_{22} = 1$.

Note that if the Hu theory is being used, the α_{ij}'s are all known constants. If the modified theory is being employed, the variable α_{ij}'s are those that have been evaluated during the preceding cycle, (k-1). This is in keeping with the overall nature of the analysis as a step-by-step procedure.

Having determined the $\bar{\sigma}_N^{(k)}$'s, one can now go to the curves representing the α_{ij}'s as functions of $\bar{\sigma}$ to evaluate $\alpha_{11}^{(k)}$, $\alpha_{12}^{(k)}$ and $\alpha_{44}^{(k)}$. After having also determined $\bar{\epsilon}_N(p)^{(k)}$ and $\Delta\bar{\epsilon}_N(p)^{(k)}$ as before by the constant strain method, these quantities may be substituted into a finite equivalent of Equations 12 specialized for the biaxial case, to yield

$$\Delta\epsilon(p)_{3N-2}^{(k)} = \frac{\Delta\bar{\epsilon}(p)_N^{(k)}}{\bar{\sigma}_N^{(k)}}\left[\alpha_{11}^{(k)}\,\sigma_{3N-2}^{(k)} - \alpha_{12}^{(k)}\,\sigma_{3N-1}^{(k)}\right]$$

$$\Delta\epsilon(p)_{3N-1}^{(k)} = \frac{\Delta\bar{\epsilon}(p)_N^{(k)}}{\bar{\sigma}_N^{(k)}}\left[\sigma_{3N-1}^{(k)} - \alpha_{12}^{(k)}\,\sigma_{3N-2}^{(k)}\right] \qquad (20)$$

$$\Delta\epsilon(p)_{3N}^{(k)} = \frac{\Delta\bar{\epsilon}(p)_N^{(k)}}{\bar{\sigma}_N^{(k)}}\left[3\alpha_{44}^{(k)}\,\sigma_{3N}^{(k)}\right]$$

From these, the strain components $\epsilon(p)_{3N-2}^{(k)}$, $\epsilon(p)_{3N-1}^{(k)}$ and $\epsilon(p)_{3N}^{(k)}$ are obtained by addition, as before, using Equation 8.

After incrementing the applied load the sequence can now be repeated for the next load cycle.

E. Rotation of Axes of Anisotropy

Anisotropic symmetry may occur in a structure for which it is convenient to choose coordinate axes that are rotated from the orthogonal axes of anisotropy. The corresponding expressions for the incremental flow equations and effective stress equations are derived in Appendix IV. The derivation is limited to the case of biaxial stress where the 3-3 and z-z axes coincide. A method of obtaining the shear anisotropic coefficients is also indicated.

F. Anisotropic Analysis of Shear Lag Structure

It has been pointed out earlier that the 2024-T4 aluminum alloy of the shear lag structure tested for the Air Force (Reference 7) displayed considerable anisotropy as shown in Figure (11). The structure had been analyzed for isotropic strain hardening based upon the curve R02M in this figure, and the results discussed in Section III. A corresponding anisotropic strain hardening analysis has also been carried out, employing first the Hu theory and then the proposed extension.

Material stress-strain data is available along two of the axes of anisotropy, the rolling or x-x (1-1) direction, and the long transverse or y-y (2-2) direction. This data has been plotted and discussed in connection with Figure (11); it is replotted for convenience in Figure (37).

The additional required, but missing, test data is (a) tensile stress-strain data in the short transverse or z-z (3-3) direction and (b) shear stress-strain data in the x-y plane. A reasonable assumption to make for engineering purposes for (a) is that the long and short transverse properties are identical; this is made in the analyses to follow. In the case of (b), the missing shear data, the following is done. First, a shear stress-strain curve is obtained based upon the tensile curve in the rolling direction, together with the assumption that the material is isotropic and governed by the incremental theory of Section III -- specifically, Equations 5 and 7. Next, a similar shear stress-strain curve based upon the tensile curve in the long transverse direction is obtained. Finally, a faired-in average of these two curves is taken to represent the missing shear stress-strain relation.

In order to apply the Hu theory, one must first select four characteristic stresses to represent the directional stress-strain curves, as discussed in Section V-B. These are the quantities Y_{11}, Y_{22}, Y_{33} and Y_{12} of Equations 13. We arbitrarily choose the proportional limits from the four curves just discussed for these values; they are

$$Y_{11} = 51 \text{ ksi}$$

$$Y_{22} = 32 \text{ ksi}$$

$$Y_{33} = 51 \text{ ksi}$$

$$Y_{12} = 22 \text{ ksi}$$

Since the simple directional stress-strain curve in the 2-2 direction, i.e. R02M, is selected for the effective stress relationship, the effective stress characteristic value K is also 32 ksi. Substituting these values in Equations 13, one can solve for the necessary anisotropic parameters. This information is all that is required to carry out an analysis of the shear lag structure by the Hu theory.

As pointed out previously, once the anisotropic parameters α_{11}, α_{22}, α_{12} and α_{44} have been specified and one of the stress-strain curves chosen as the effective stress-strain curve, the remaining simple direc-

tional stress-strain curves are prescribed. Let us now examine the consequences of our current parameter selection. Since only test data in the 1-1 and 2-2 directions are available, we shall concentrate on these. For this special case, Equations 11 and 12 can be easily manipulated to yield

$$\bar{\sigma} = \sqrt{\alpha_{11}}\,\sigma_{11}$$

$$\frac{d\epsilon_{11}}{d\sigma_{11}} = \alpha_{11}\frac{d\bar{\epsilon}}{d\bar{\sigma}} \qquad (21)$$

Using these expressions, one can construct a σ_{11} stress-strain curve.

Such a curve is shown on Figure (37). As can be seen, the fit with the 1-1 test data is very poor. Apparently, one could do better by arbitrarily choosing for the 1-1 characteristic value Y_{11} a much lower value than 51 ksi -- perhaps in the neighborhood of 35 ksi. Nevertheless, it is clear that in this case it would still be impossible to get a really good fit because of the fundamentally different shapes of the two stress-strain curves, especially in the region of their knees. It will turn out that, in the case of the shear lag structure, this selection is not critical, because of the fact that stresses in the 1-1 direction are very low, compared to those in the 2-2 direction.

In order to carry out an analysis of the shear lag structure based upon the extension to the Hu theory, one must first evaluate the plastic work done in each of the simple directional stresss-strain tests as a function of the applicable stress. Using this information in the manner discussed previously, one can then obtain the anisotropic parameters as functions of effective stress. This has been done, assuming that it is sufficiently accurate to represent the curves by a small number of connected straight line segments. The key values of the resulting α's are given in the accompanying table. Corresponding total strains $\bar{\epsilon}$ ($= \bar{\sigma}/E + \bar{\epsilon}$) are also listed.

$\bar{\sigma}$	$\bar{\epsilon}$	$\alpha_{11} = 2\alpha_{12}$	α_{22}	α_{44}
32000	.00310	0.394	1.000	0.697
37000	.00378	.510	1.000	.755
40000	.00435	.590	1.000	.795
43000	.00509	.660	1.000	.833
46000	.00622	.742	1.000	.871
48000	.00734	.809	1.000	.905
49000	.00800	.835	1.000	.918
51000	.00980	.892	1.000	.945
54000	.03800	.930	1.000	.965

In the preceding table, the reduction in number of independent anisotropic parameters is a consequence of the assumptions which had to be made for the missing stress-strain curves.

The value of unity for α_{22} is in accordance with our continued selection of the tensile stress-strain curve R02M in the y-y or 2-2 direction as the basis for our effective stress-strain relationship.

It might be reiterated that, because of the manner in which the α's are derived in this case, all of the simple directional stress-strain curves are matched as closely as desired.

Using first the anisotropic parameters selected as described for the Hu theory, and then for its extension, the shear lag structure has been analyzed. The results are presented in the next section.

G. Discussion of Results

Some of the highlights of the two anisotropic analyses are presented in Figures (38) through (40). Corresponding results obtained previously and based upon isotropic theory are also included for comparison.

Figure (38) refers to the central node of the shear lag structure and shows the variation of the total strains in the x and y directions with increasing applied load. The isotropic result is replotted from Figure (22) -- specifically those curves based upon the effective stress-strain curve R02M.

It can be seen from Figure (38) that the most flexible analysis predictions, that is, those for which the total strains are the largest, are obtained by the isotropic analysis. The analysis based upon the extension to the Hu theory is somewhat less flexible, while the Hu theory results are the stiffest. Also, the differences between the three analyses are less in the y direction than in the x direction.

These observations are clearly in agreement with the nature of the material stress-strain relations upon which the analyses are based. In all three cases, we are assuming that the y-y curves are identical, that is, all are represented by R02M. Referring to Figure (37), in the case of the isotropic analysis, we are also assuming, in effect, that the x-x stress-strain curve is identical to the y-y curve R02M. The x-x curve for the extension to the Hu theory is seen to be somewhat stiffer than R02M, while the x-x curve for the Hu theory is by far the stiffest of the three.

Comparing the analyses with the test data, all three analyses substantially underpredict the test points in the high applied load regime, and more so in the y than in the x direction. It is interesting to note however, that while the anisotropic analyses make these differences even greater, they do have the virtue of making the comparison more consistent as between the x and y directions. Thus it would appear that the

anisotropic nature of the 2024-T4 material does influence the strain distribution in the shear lag structure, and that the anisotropic analyses can detect this tendency.

Figures (39) and (40) show the strain distributions along the two strain gage lines of the test structure. In addition to the isotropic predicted results, replotted from Figures (20) and (21), anisotropic results based upon the extension to the Hu theory are presented. They indicate that while the differences are not dramatic, they do in fact exist.

SECTION VI

ANISOTROPIC ELASTIC-PLASTIC-CREEP ANALYSIS

A. Anisotropic Creep Theory

The application of matrix analysis procedures to problems involving anisotropic creep is at present academic. Neither meaningful analytical research nor appropriate test data has been found. Therefore the procedure presented here for anisotropic creep is a simple extension of that already described in previous sections. Accordingly, only the difference and additional assumptions are discussed.

Strains due to anisotropic creep can be handled in a way similar to those due to isotropic creep. The step-by-step procedure has only to be modified for the anisotropic behavior by substituting Equations 19 and 20 for Equations 5 and 7 respectively in the manner described for time independent plastic anisotropy in Section V-D. The additional assumptions implied by this simple extension are the following:

(1) The anisotropic parameters calculated from zero time simple directional tests (by either Hu theory or the proposed extension) are valid for anisotropic creep.

(2) The effective creep strain equation, Equation 10, remains valid and the empirical constants (α,β,γ) are determined for the tensile creep test in the assumed effective stress-strain direction.

Because testing of structures exhibiting anisotropic creep has apparently not been done, the anisotropic creep procedure cannot be checked out against tests at this time. However, the results of a sample calculation are presented for a hypothetical material having this characteristic.

B. Sample Problem

The 1100-F aluminum shear lag specimen, already analyzed for isotropic creep, is used for an anisotropic creep analysis making the following assumptions:

(1) All the uniaxial data (Figure 25) employed in the isotropic creep analysis is assumed to refer to the y-y (2-2) effective direction of the anisotropic creep analysis.

(2) The plastic anisotropic parameters (extension of Hu's theory) for the effective stress-strain curve of the 2024-T4 aluminum alloy material for successive levels of stress, 32 ksi, 37 ksi, 40 ksi, etc., are arbitrarily chosen for the stress levels, 2 ksi, 3 ksi, 4 ksi, etc., of the 1100-F aluminum effective stress-strain curve TC1 (Figure 25). Thus:

$\bar{\sigma}$	$\bar{\varepsilon}$	$\alpha_{11} = 2\alpha_{12}$	α_{22}	α_{44}
2000	.000222	0.394	1.000	0.697
3000	.000359	.510	1.000	.755
4000	.000540	.590	1.000	.795
5000	.000815	.660	1.000	.833
6000	.001256	.742	1.000	.871
7000	.001967	.809	1.000	.905
8000	.003038	.835	1.000	.918
10000	.007110	.892	1.000	.945

This means that the uniaxial x-x (1-1) predicted stress-strain curve and the shear x-y stress-strain curve are arbitrarily stiffer than the corresponding isotropic curves.

The node strains obtained, using the anisotropic assumptions for load P = 2020 lbs at time t = 3 hrs, are presented for the x-axis and along x = 1 in. in Figures (41) and (42) respectively.

In general, the anisotropic results appear to be stiffer than those for the isotropic case, and this is especially true for the vertical gage line, where the anisotropic shear strains are of the order of 3/4 of the isotropic values. This should be expected, due to the fact that the anisotropic parameters used are all less than, or at the most equal to, their isotropic equivalents.

SECTION VII

CONCLUSION

The linear, matrix structural analysis methods currently in general use throughout the aerospace industry have recently been extended to include structures loaded into the inelastic material behavior regime. However, very little published information is available correlating predicted results with the test data.

The current report recommends a simple analytical approach to such problems. It is based upon the concept of initial strains in combination with a suitable matrix of influence coefficients, obtained by standard linear matrix structural analysis methods. The initial strains are those associated with plasticity and creep.

Some particularly useful tests of aluminum and aluminum alloy shear lag structures have been performed previously for the Air Force. These structures have been analyzed by the recommended method, and the resulting agreement (for both the plasticity and the creep tests) is considered to be very encouraging. On the other hand, additional testing must be carried out and correlations with analysis made before the method can be considered as fully evaluated.

In the meantime, the writers believe that sufficient confidence in the method has been established that it may now be used in practical engineering applications. For example, it should be immediately useful in such problems as predicting inelastic strain distributions around stress raisers in simple structural components.

As for the influence of anisotropy, it has been shown that this material property can be readily accommodated in the recommended procedure. Calculations made for the 2024-T4 shear lag structure indicate that, in this case at least, anisotropy plays a minor but discernable role in determining the strain distributions. This evidence is inconclusive, because in this particular test, while the material itself is decidely anisotropic, the stresses normal to the direction of the applied loads are quite small.

It is recommended that in cases where doubt exists as to the importance of anisotropy, it be included in the analysis. Certainly in such examples as the shear lag structure, the additional complexity in the use of the computer program is very slight.

There is one very important restriction implicit in the method proposed in this report. Essentially, this method applies only to a structure in which the material is initially in the virgin state, and thereafter experiences only continually-increasing applied loads until failure occurs. This limitation can actually be relaxed to the extent that elastic unloading followed by reloading can also be accommodated, but

completely reversed loading is specifically excluded. For the latter case, a different plasticity theory is needed. It is important that a technique for handling such problems be developed because of the need for such applications in fatigue work. Efforts toward removing this restriction are currently under way under government contract.

Other than the preceding, the largest remaining obstacle to complete inelastic analysis of practical aerospace structures is believed to be the dearth of appropriate material property data, and constitutive laws to describe them.

SECTION VIII

FIGURES

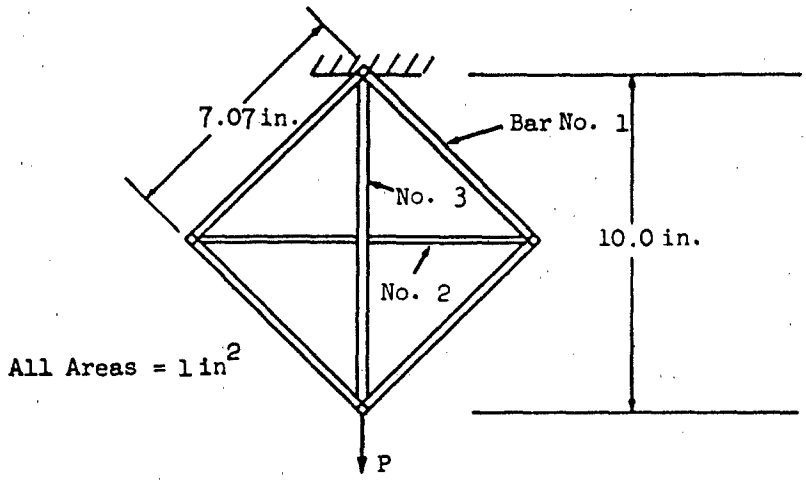


Fig. 1 Truss Structure

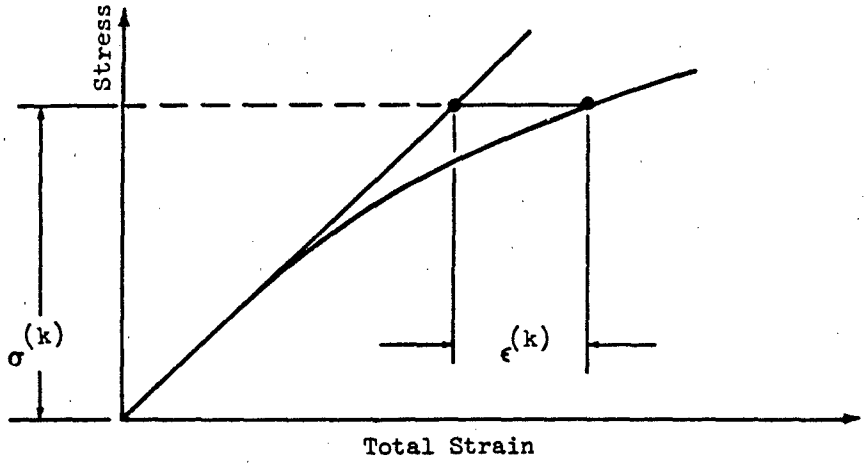


Fig. 2 Constant Stress Method

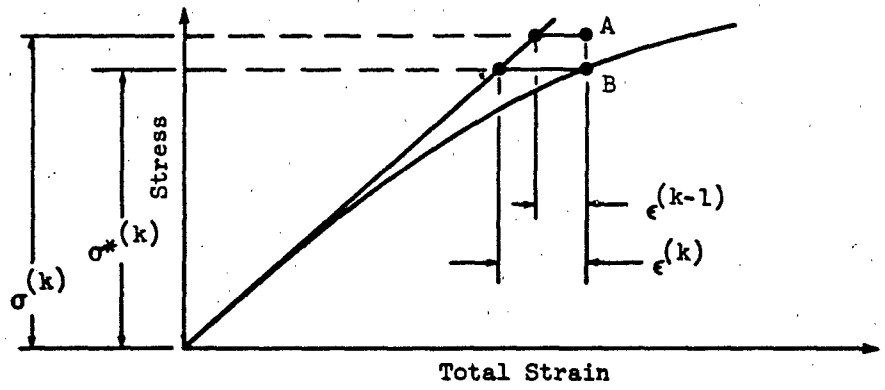


Fig. 3 Constant Strain Method

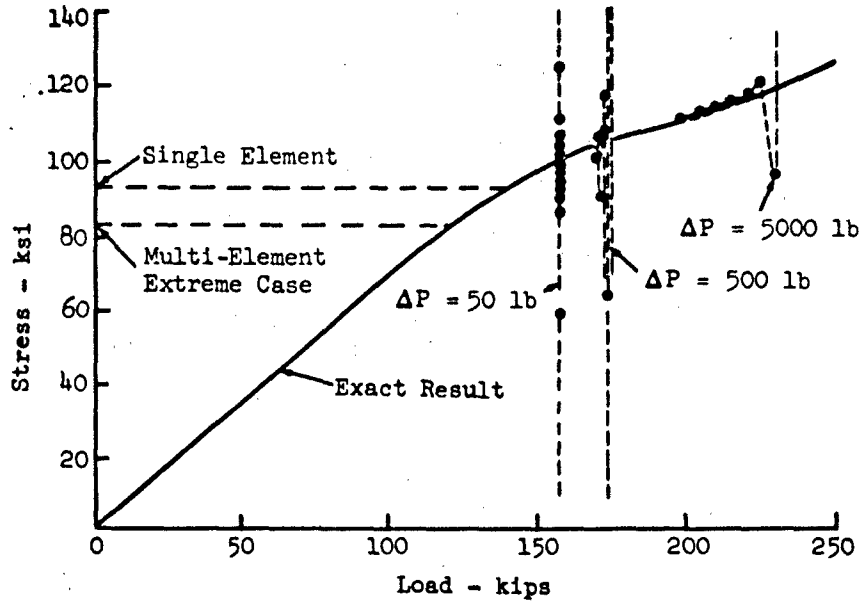


Fig. 4 Results of Constant Stress Method for Bar No. 3 of Truss

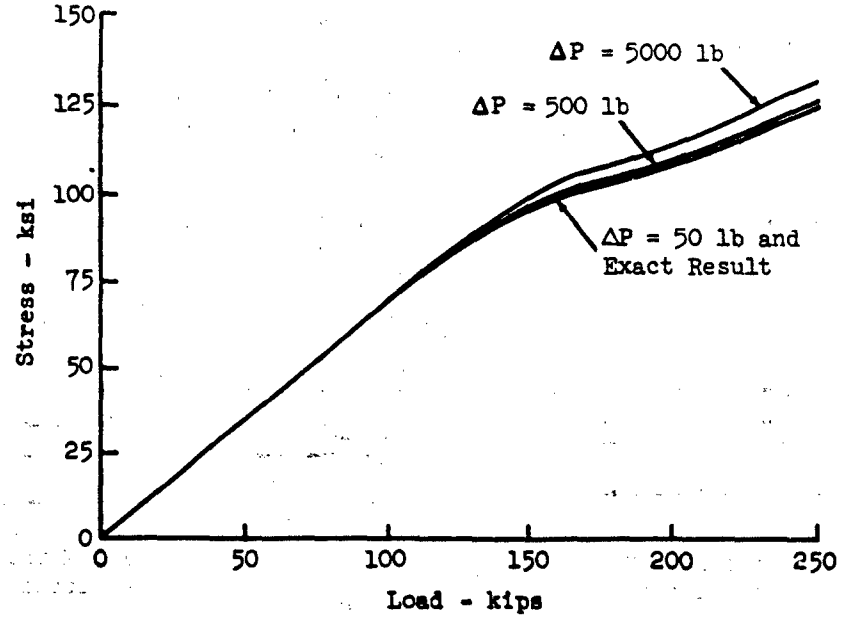


Fig. 5 Results of Constant Strain Method for Bar No. 3 of Truss

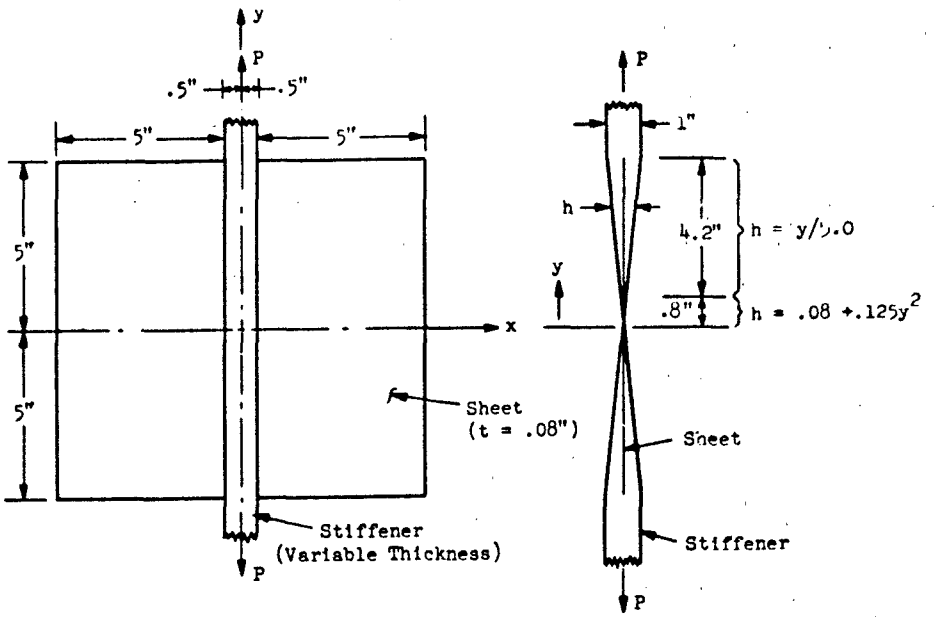


Fig. 6 Shear-Lag Specimen Designated SLS1 - From Air Force Report No. RTD-TDR-63-4032; 2024-T4 Aluminum Alloy (1100-F Specimen Similar)

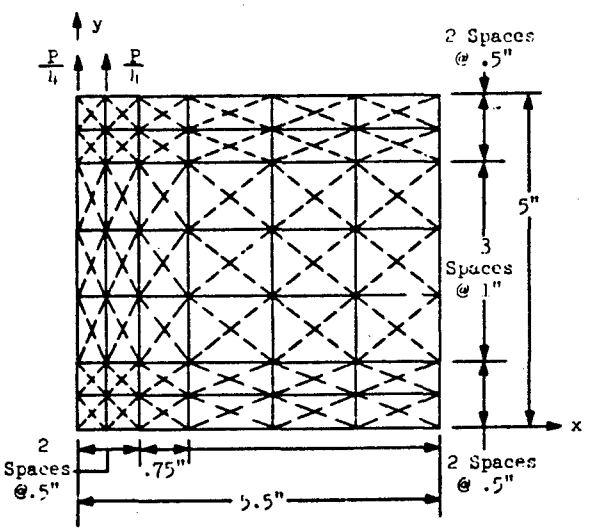


Fig. 7 Stiffness Method Idealization

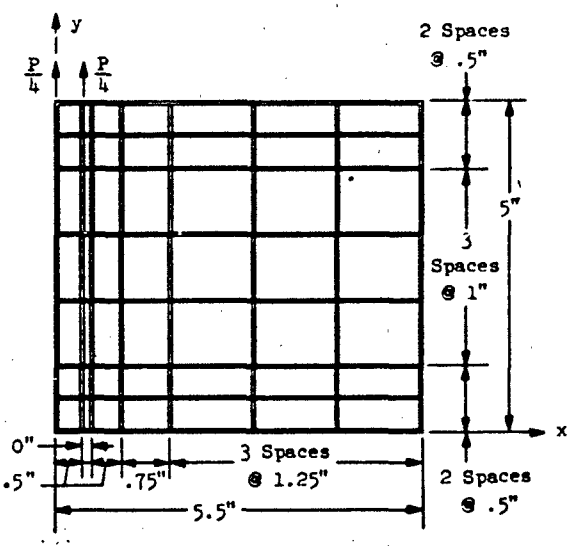


Fig. 8 Force Method Idealization

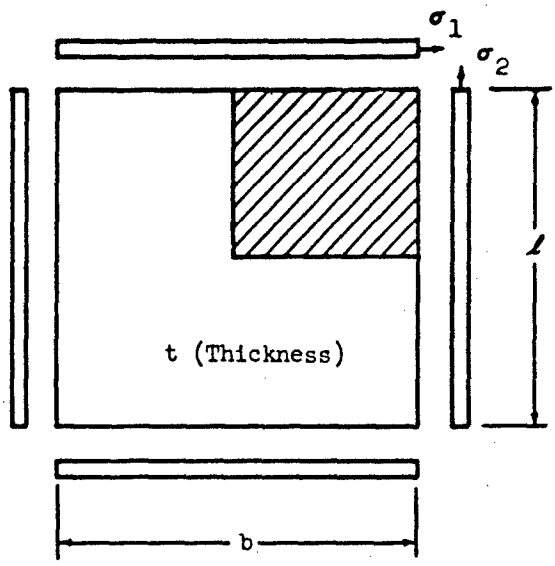


Fig. 9 Typical Elements of Force Idealization

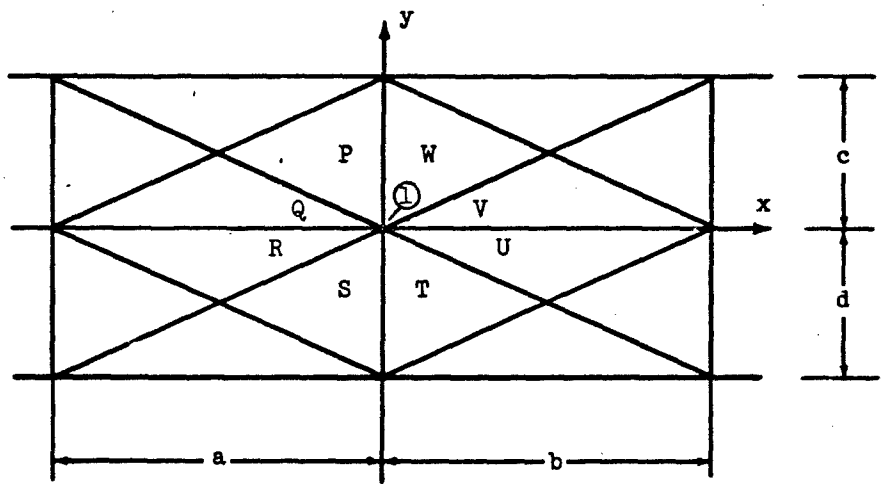


Fig. 10 Typical Elements of Stiffness Idealization

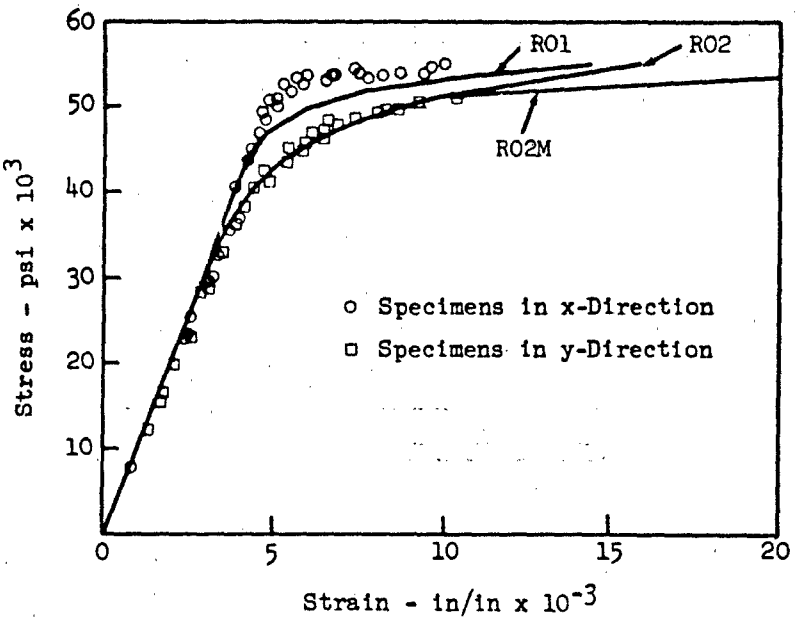


Fig. 11 2024-T4 Aluminum Alloy Stress-Strain Data and Curve R02M

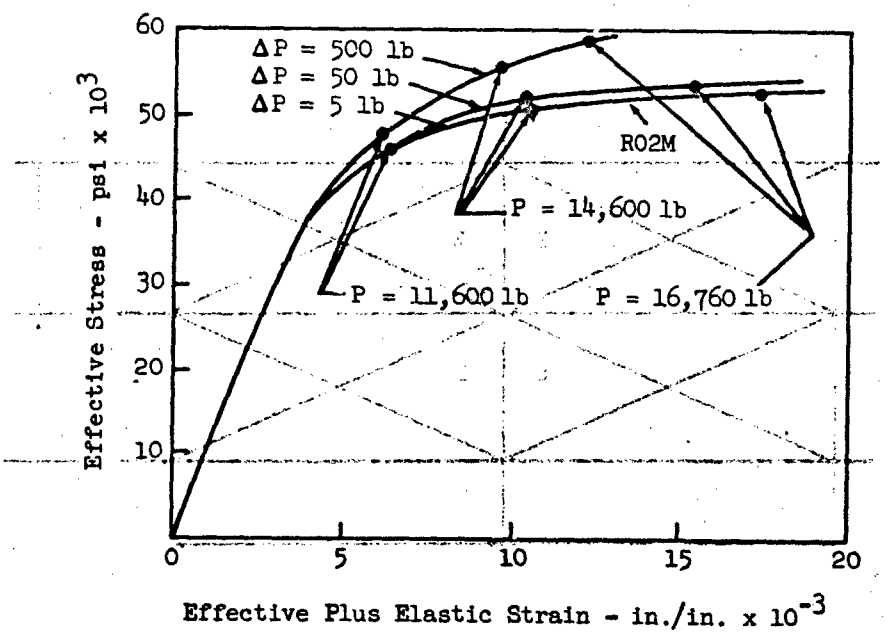
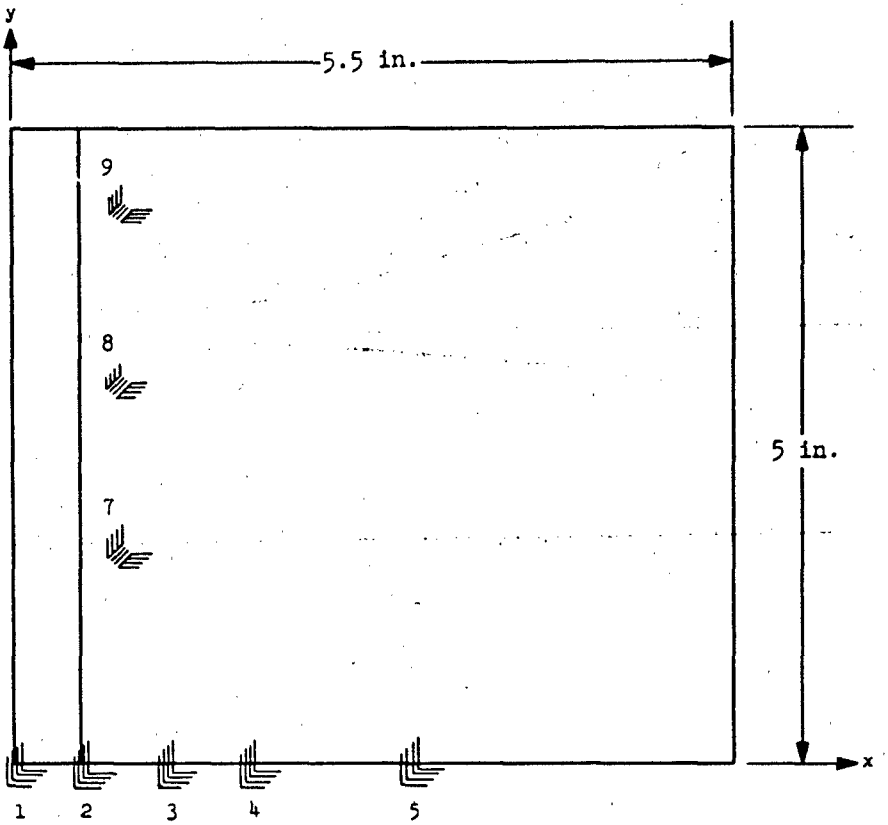


Fig. 12 Comparison of Predicted Effective Stress-Strain Relationship with R02M at (0, 0) for Various Load Increments



Gage Nos.		Co-ordinates		Gage Nos.		Co-ordinates	
Front	Back	x in.	y in.	Front	Back	x in.	y in.
1	11	0.0	0.0	6	16	-1.125	0.0
2	12	0.5	0.0	7	17	0.8125	1.5625
3	13	1.125	0.0	8	18	0.8125	2.8125
4	14	1.750	0.0	9	19	0.8125	4.0625
5	15	3.000	0.0				

Fig. 13 Instrumentation of 2024-T4 Aluminum Alloy Specimen

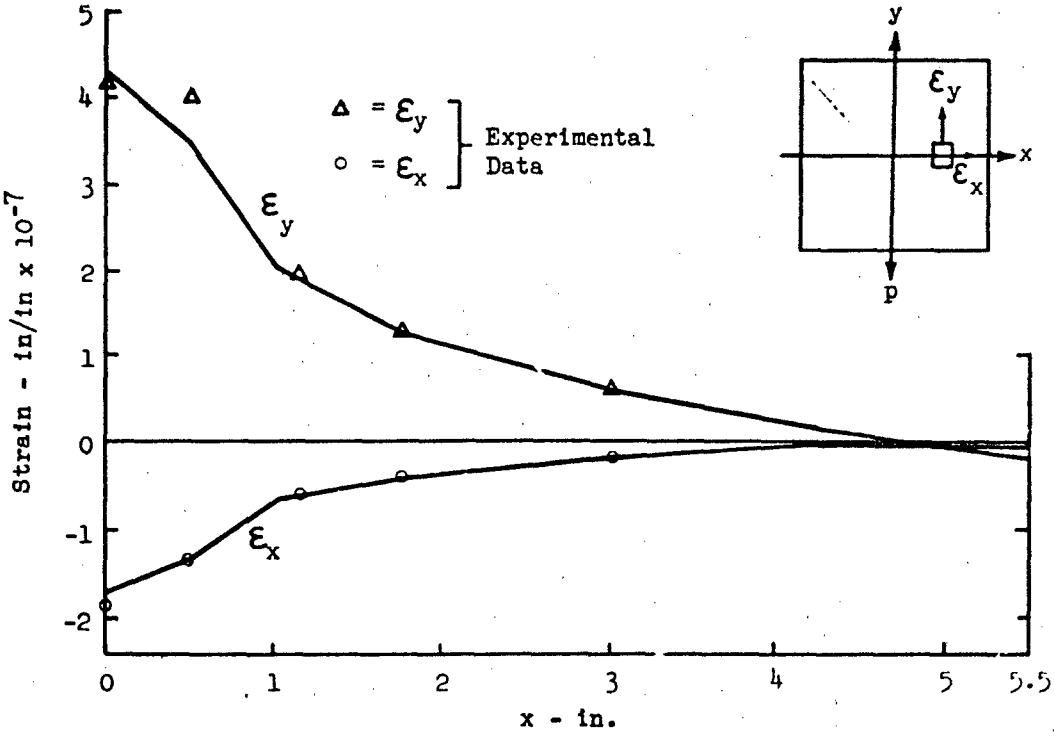


Fig. 14 Elastic Strains Along x-Axis for P = 1 lb

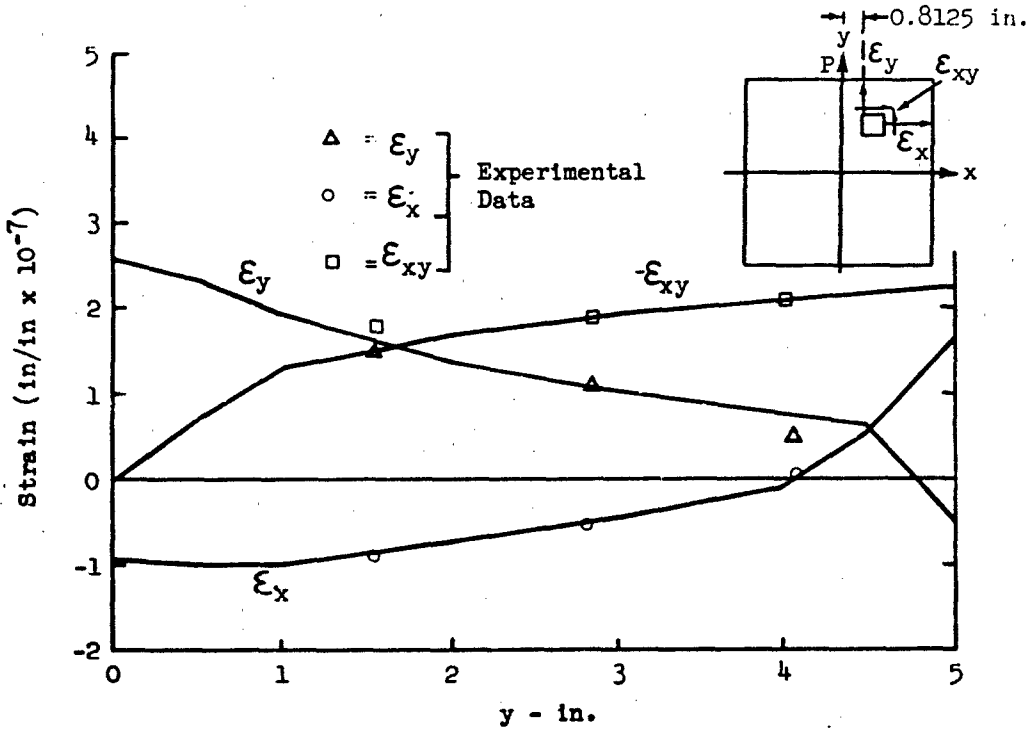


Fig. 15 Elastic Strains Along x = 0.8125 in. for P = 1 lb

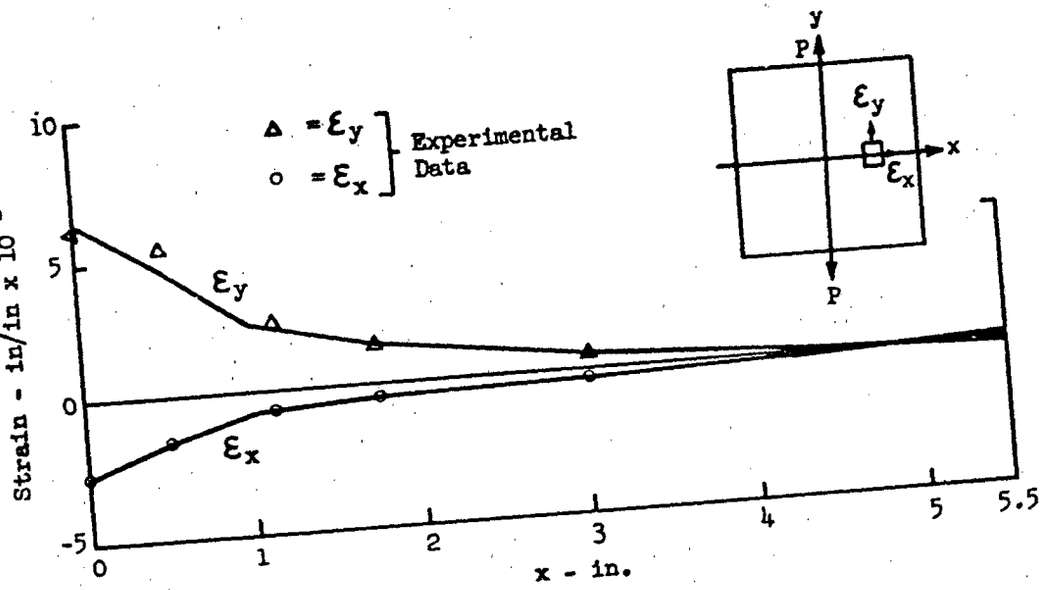


Fig. 16 Strains Along x-Axis for P = 11,600 lb and ΔP = 5 lb

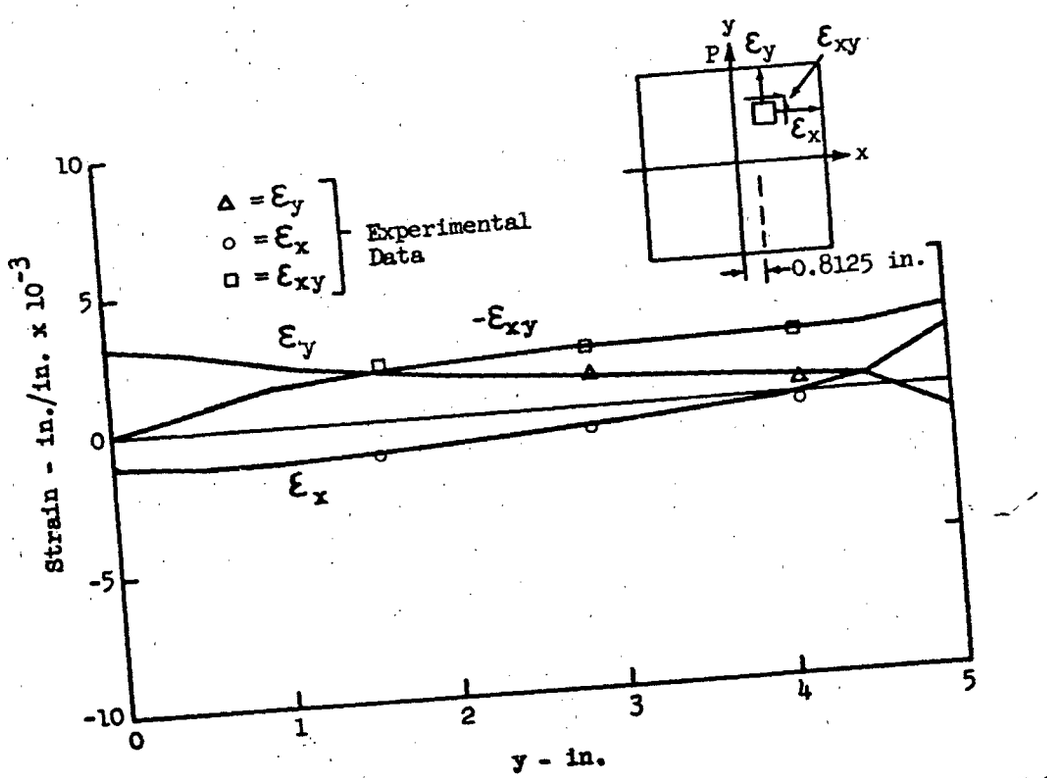


Fig. 17 Strains Along x = 0.8125 in. for P = 11,600 lb and ΔP = 5 lb

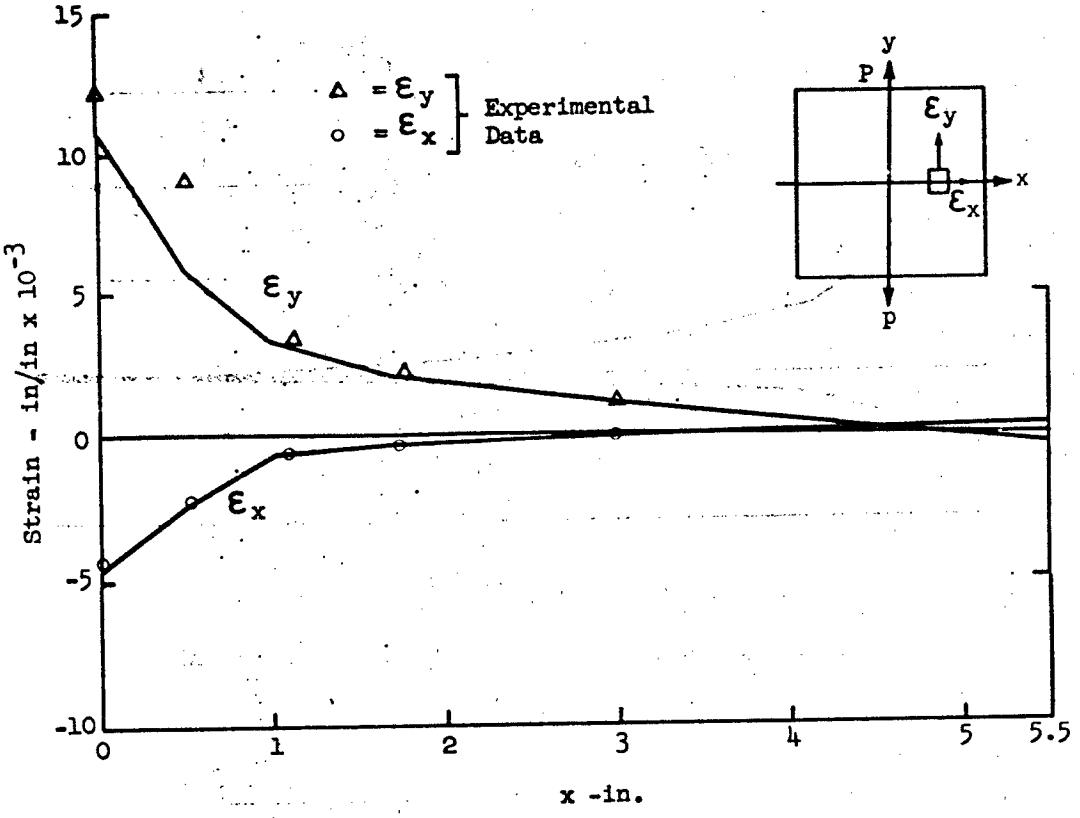


Fig. 18 Strains Along x-Axis for P = 14,600 lb and ΔP = 5 lb

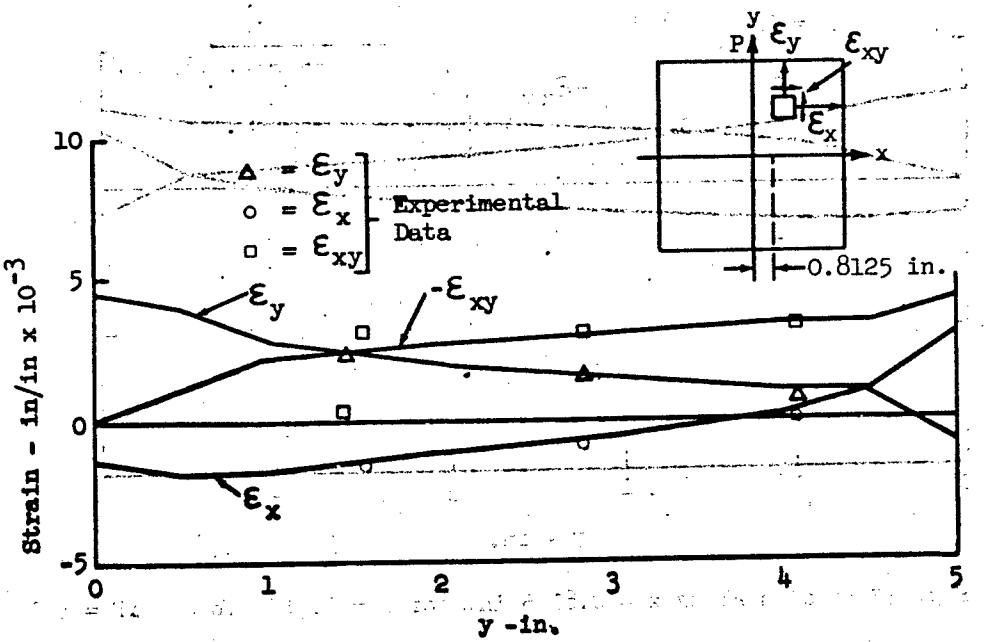


Fig. 19 Strains Along x = 0.8125 in. for P = 14,600 lb and ΔP = 5 lb

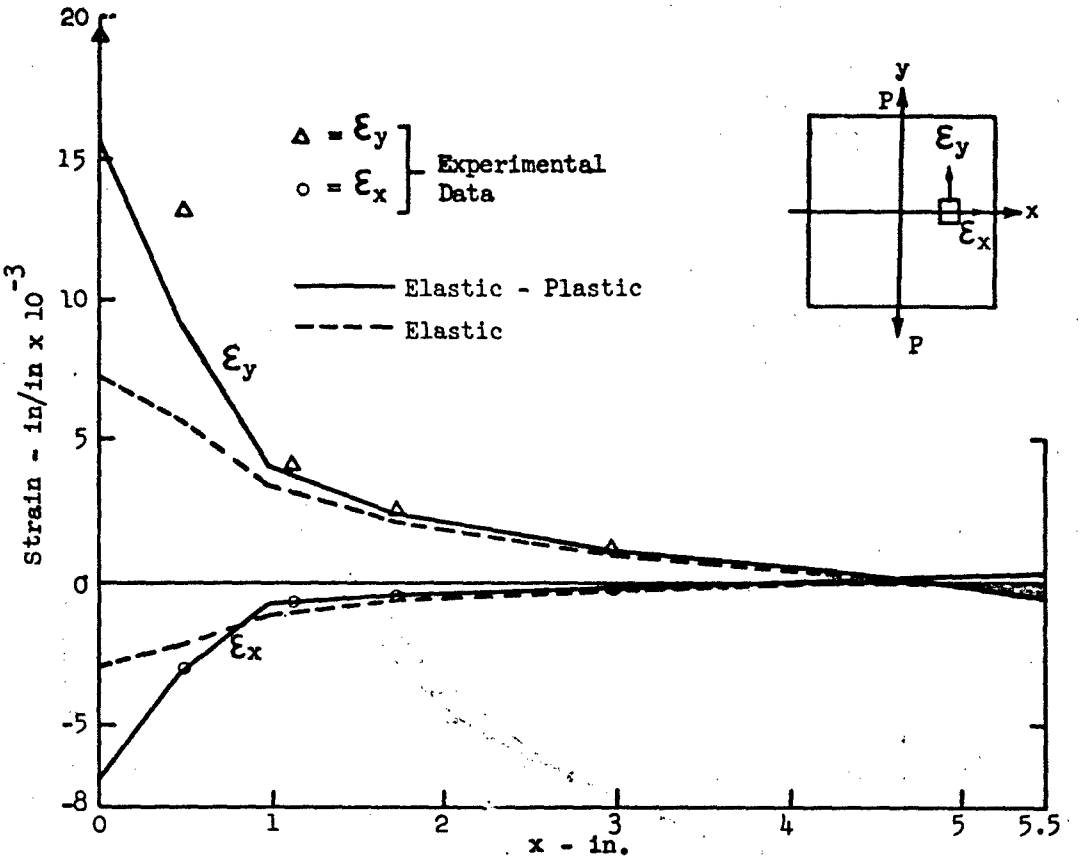


Fig. 20 Strains Along x-Axis for P = 16,760 lb, and ΔP = 5 lb

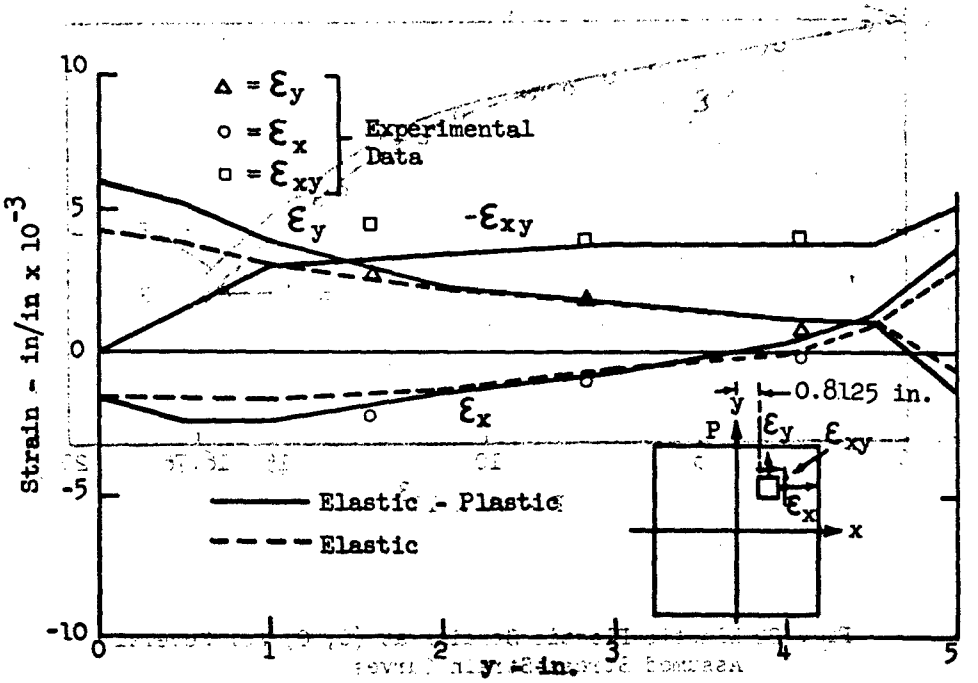


Fig. 21 Strains Along x = 0.8125 in. for P = 16,760 lb and ΔP = 5 lb

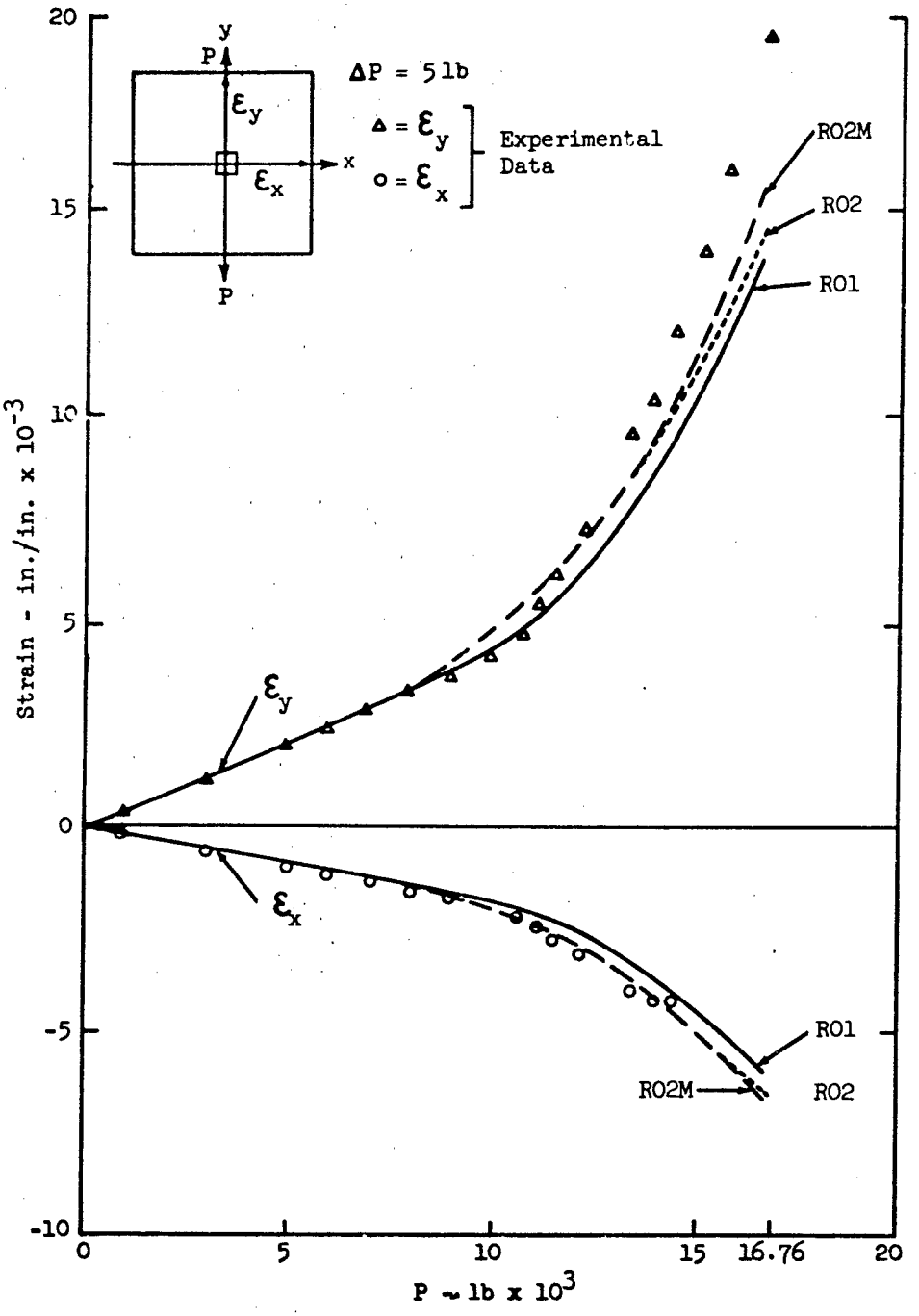


Fig. 22 Elastic-Plastic Strains at (0, 0) for Several Assumed Stress-Strain Curves

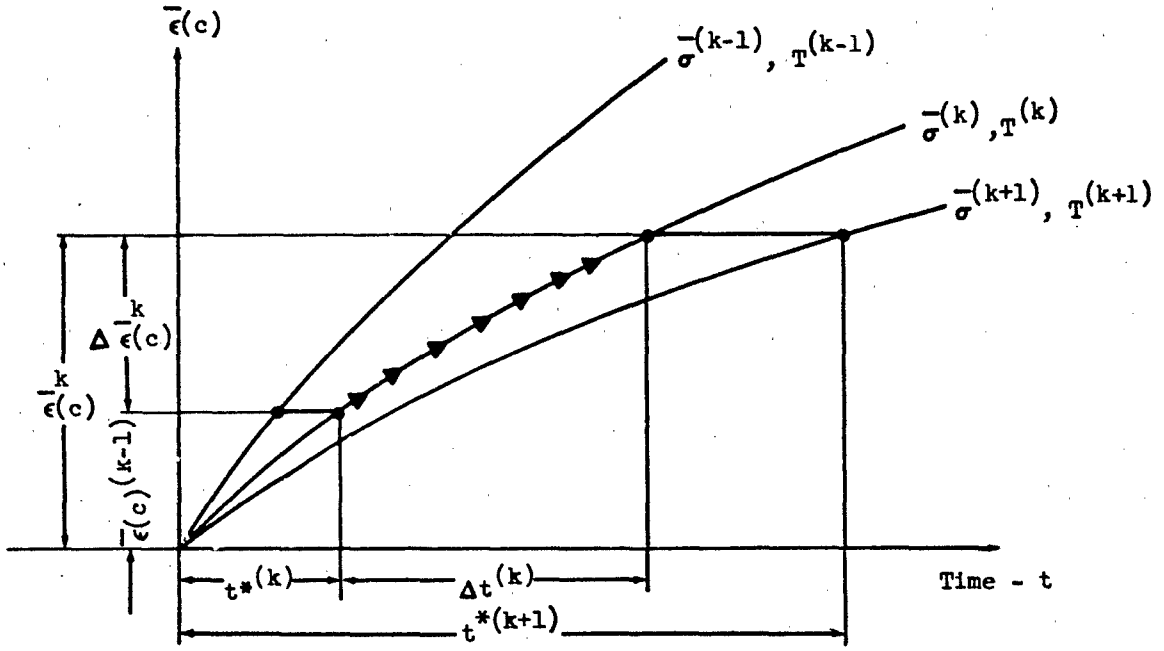


Fig. 23 The Strain Hardening Rule

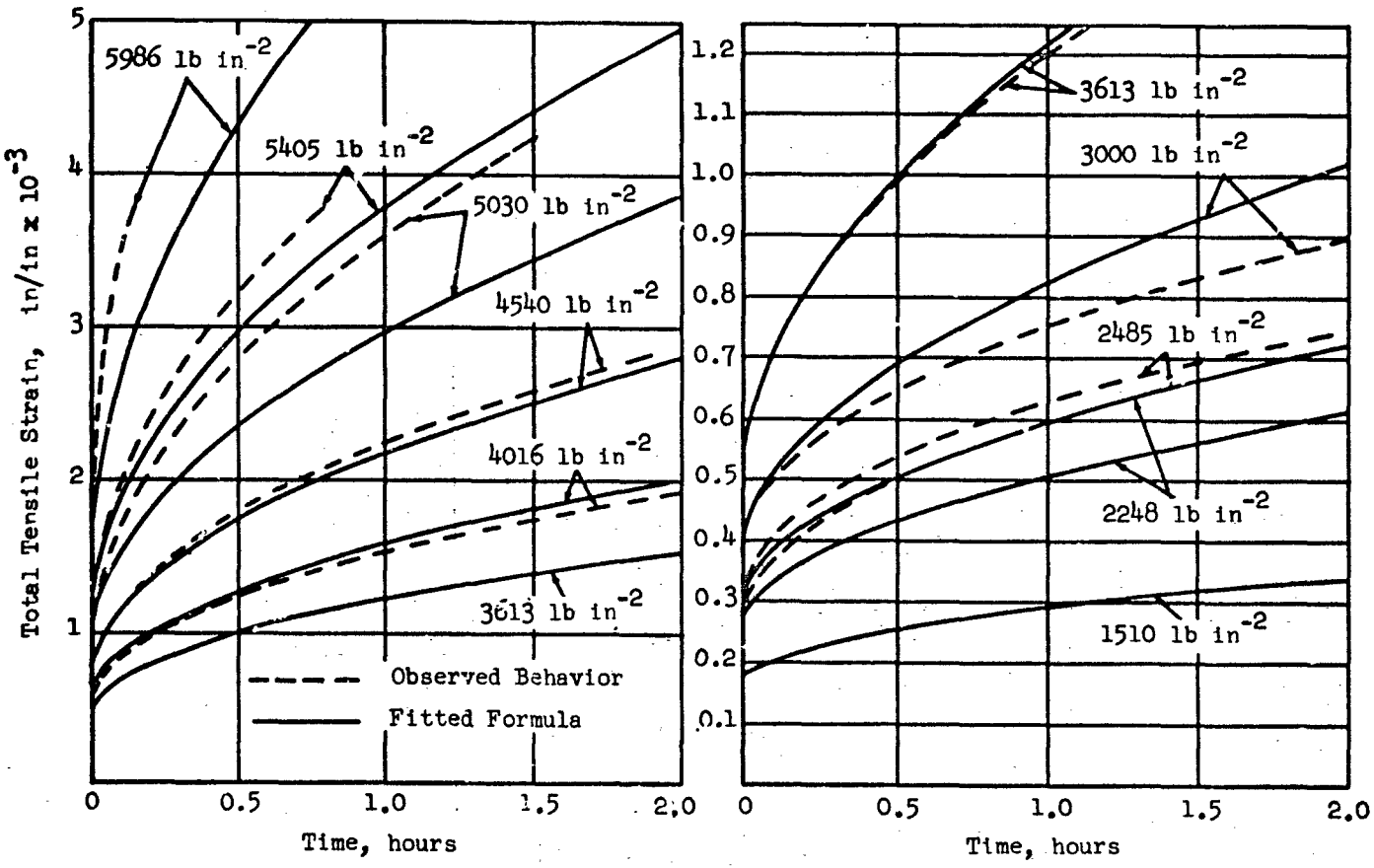


Fig. 24 1100-F Aluminum Time-Dependent Behavior and Fitted Curves (Ref. 7)

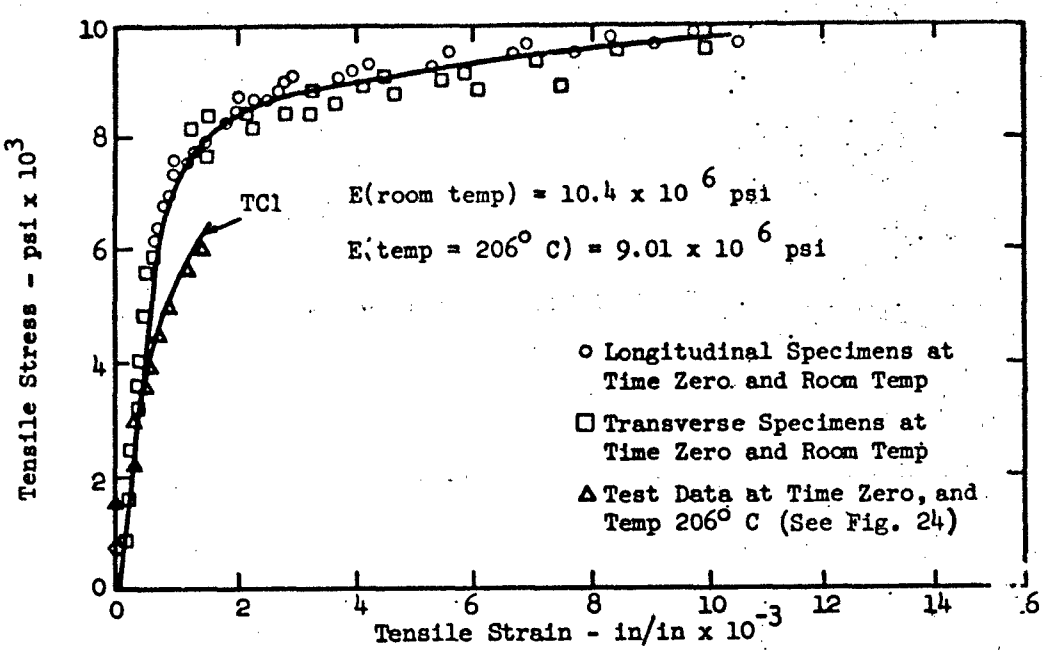
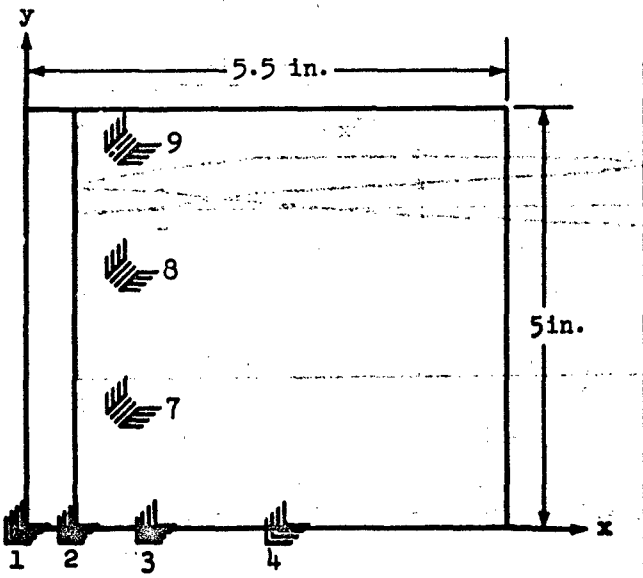


Fig. 25 1100-F Aluminum Stress-Strain Data at Room Temperature and at 206° C, for Time t = 0.00



Gage Nos.		Co-ordinates	
Front	Back	x in.	y in.
1	11	0.0	0.0
2	12	0.5	0.0
3	13	1.5	0.0
4		3.0	0.0
7		1.0	1.5
8		1.0	3.0
9		1.0	4.5

Fig. 26 Location of Strain Gages on 1100-F Specimen (Ref. 7)

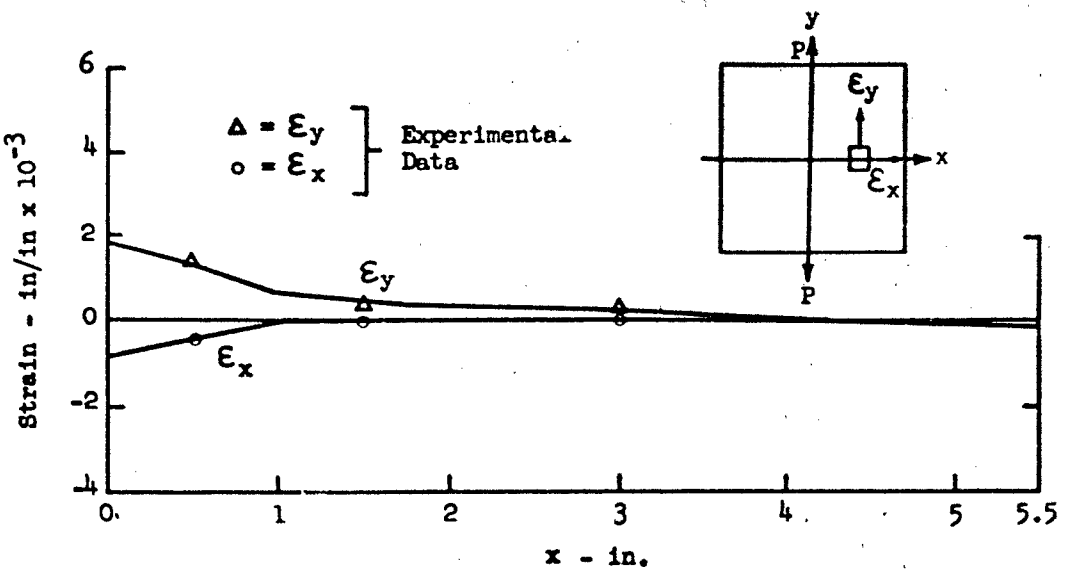


Fig. 27 Strains Along x-Axis for P = 1600 lb and t = 0.06 hr

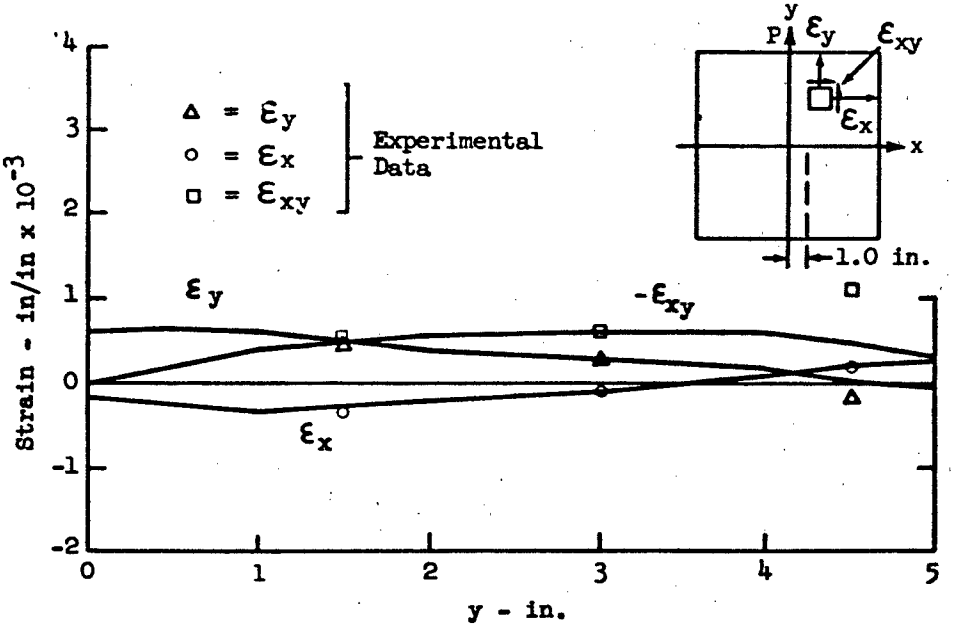


Fig. 28 Strains Along x = 1 in. for P = 1600 lb and t = 0.06 hr

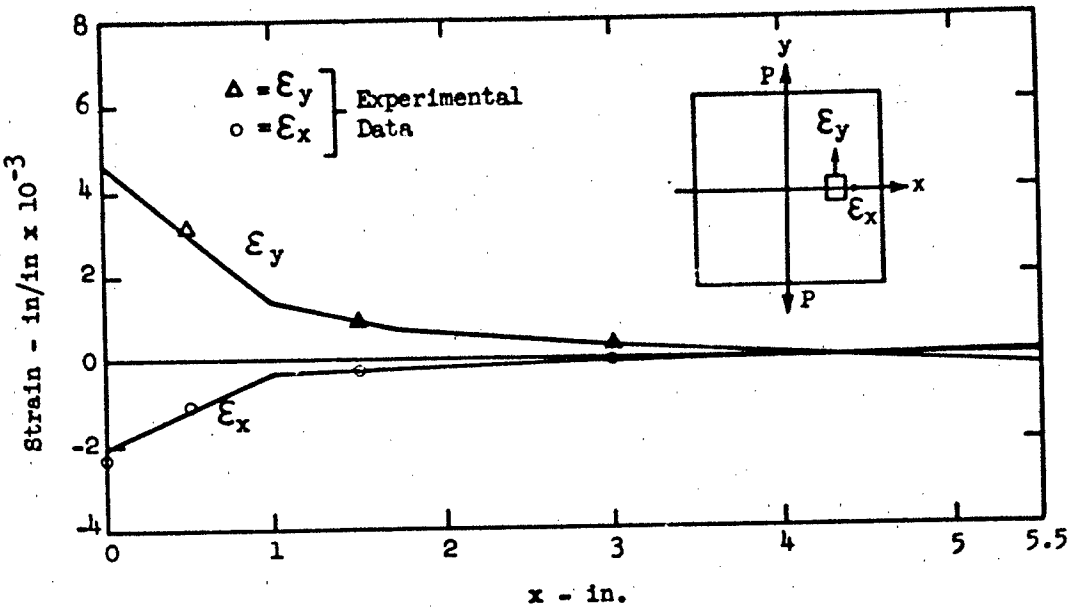


Fig. 29 Strains Along x-Axis for P = 2020 lb and t = 1.10 hr

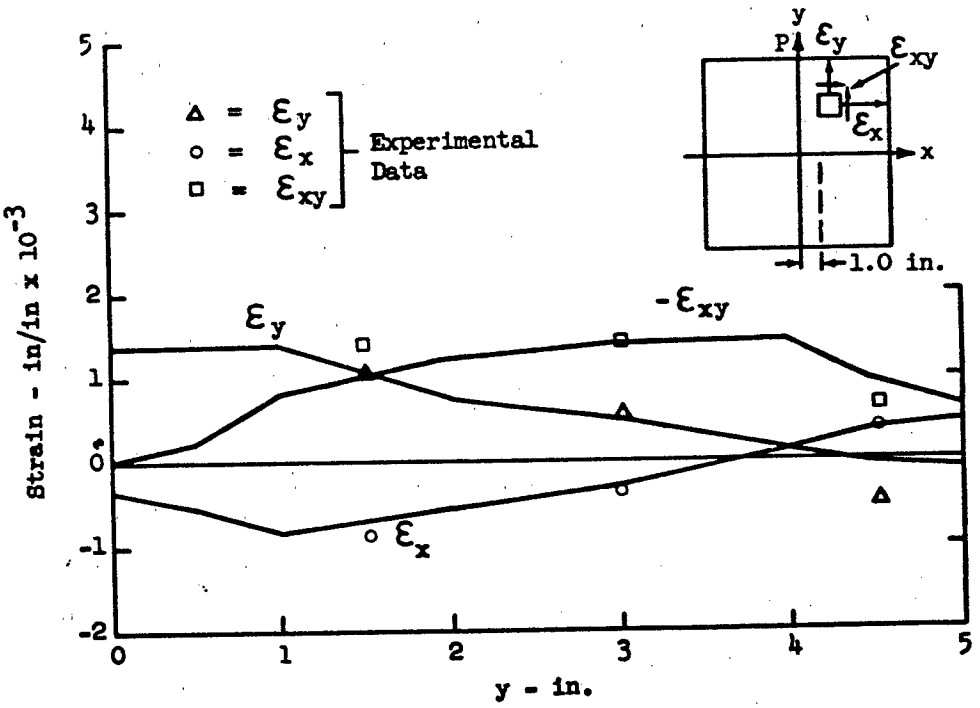


Fig. 30 Strains Along x = 1 in. for P = 2020 lb and t = 1.10 hr

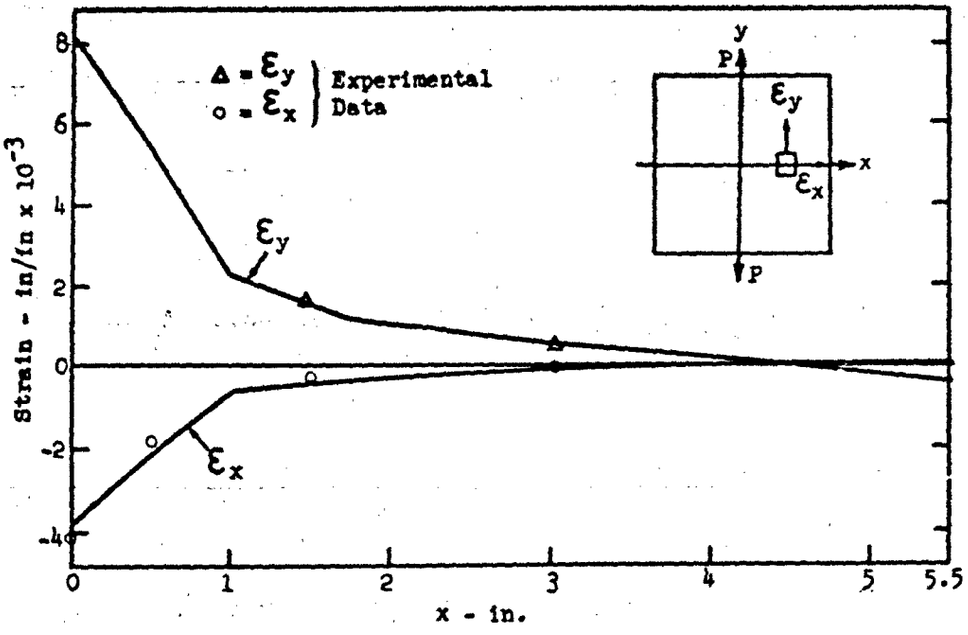


Fig. 31 Strains Along x-Axis for P = 2020 lb and t = 3.0 hr

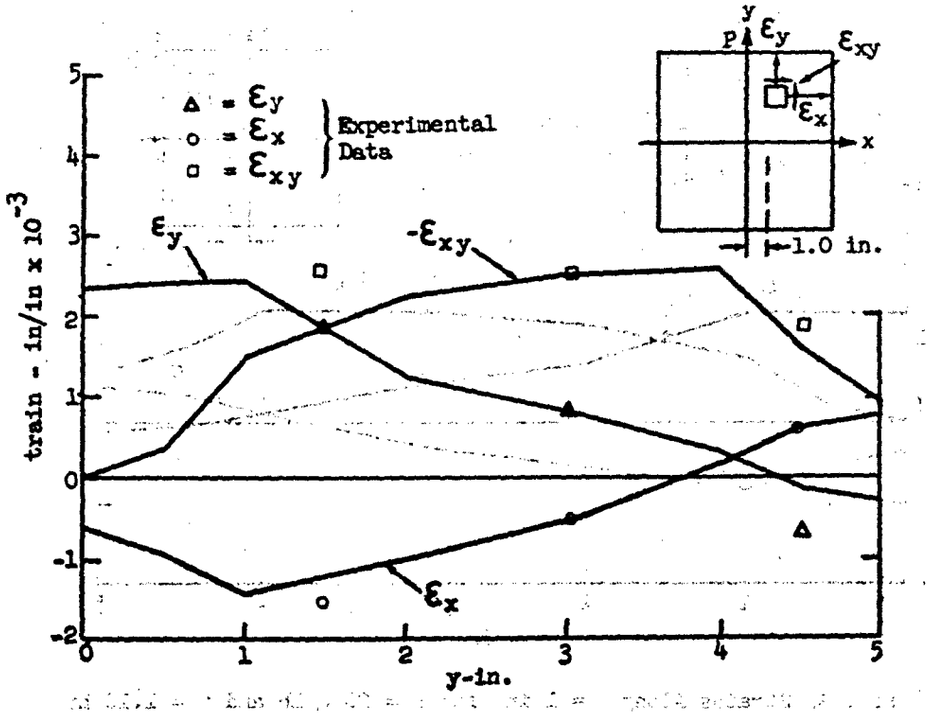


Fig. 32 Strains Along x = 1 in. for P = 2020 lb and t = 3.0 hr

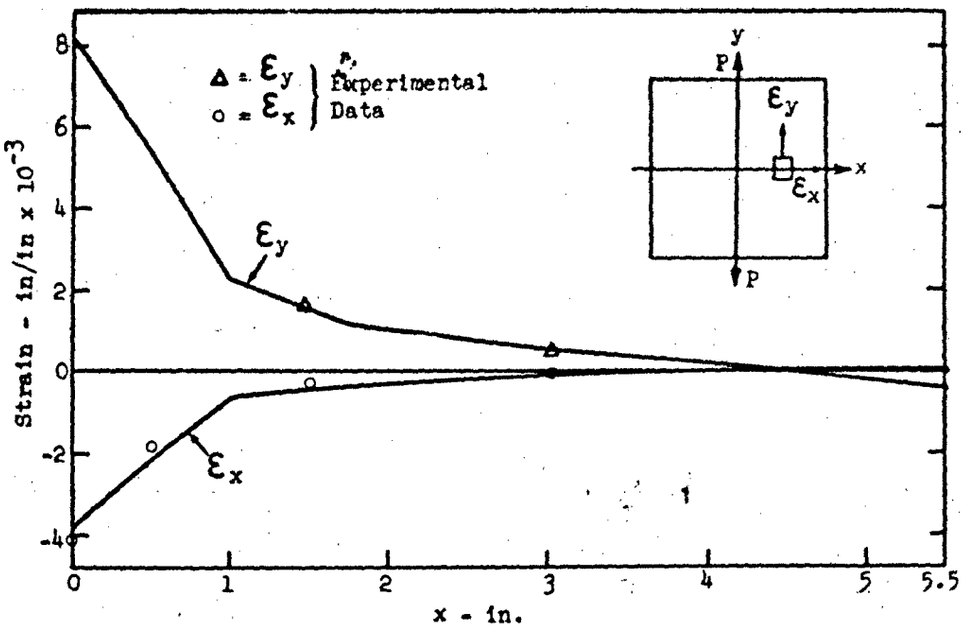


Fig. 31 Strains Along x-Axis for P = 2020 lb and t = 3.0 hr

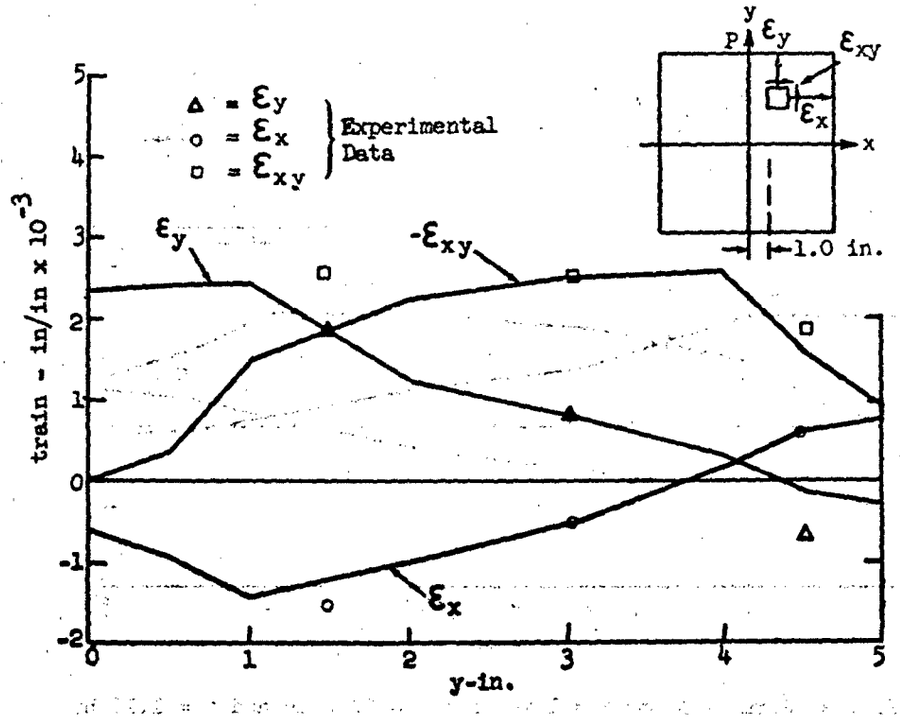


Fig. 32 Strains Along x = 1 in. for P = 2020 lb and t = 3.0 hr

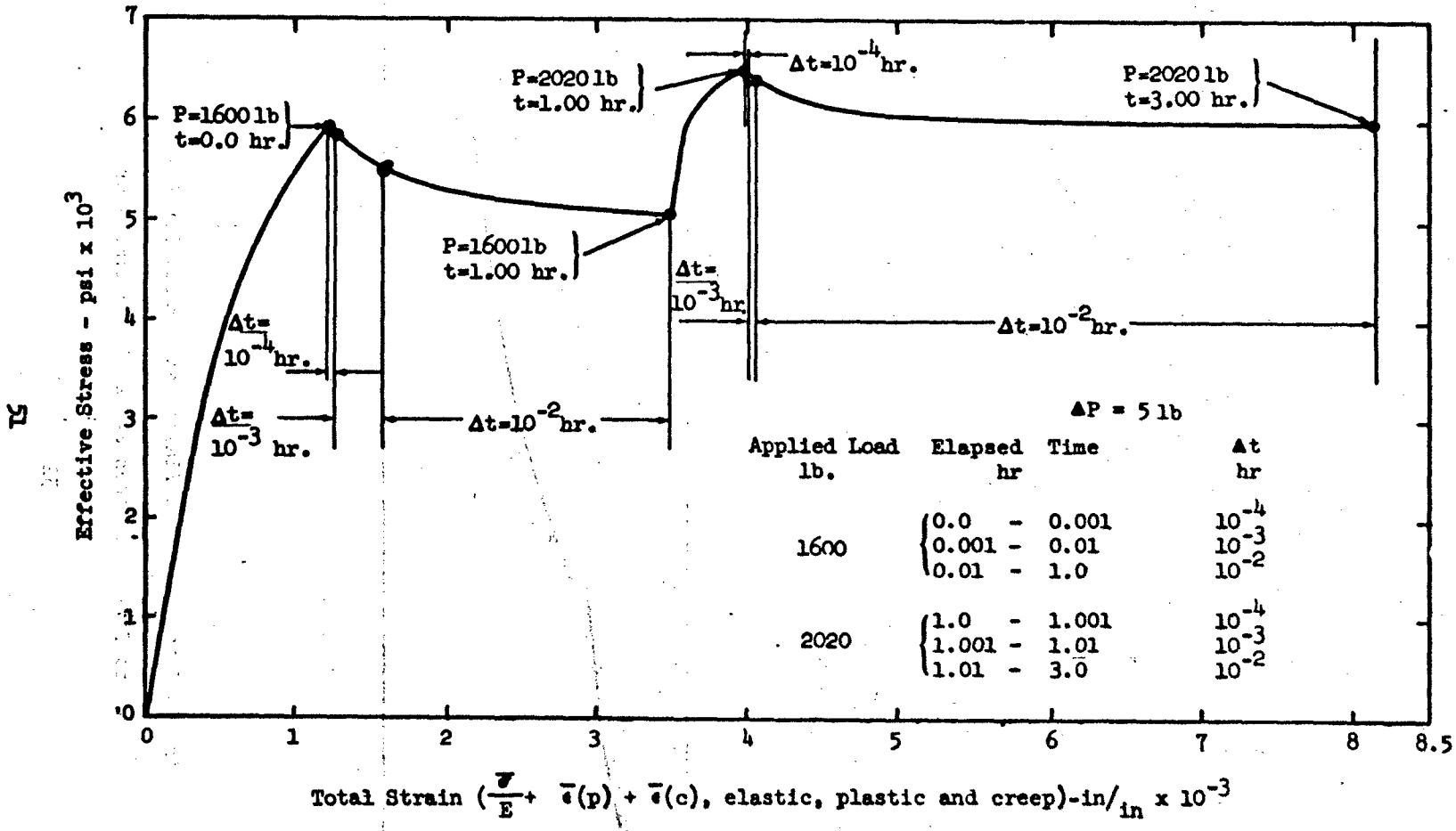


Fig. 33 Effective Stress vs Strain at Center Node

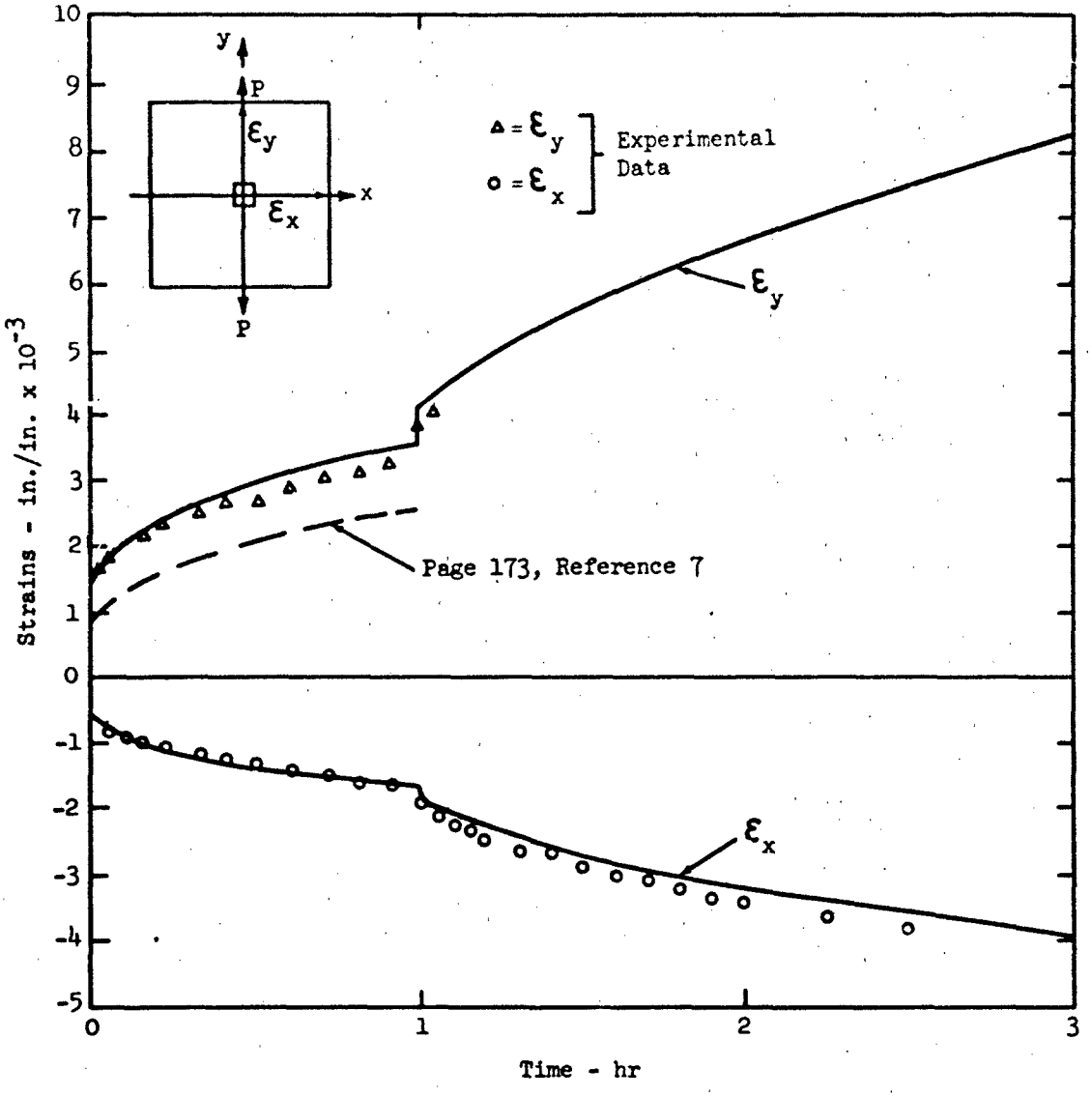


Fig. 34 Total (Elastic, Plastic and Creep) Strains at Center of Specimen (Gage 1) for P=1600 lb to Time 1 hr, then P=2020 lb to Time 3 hr

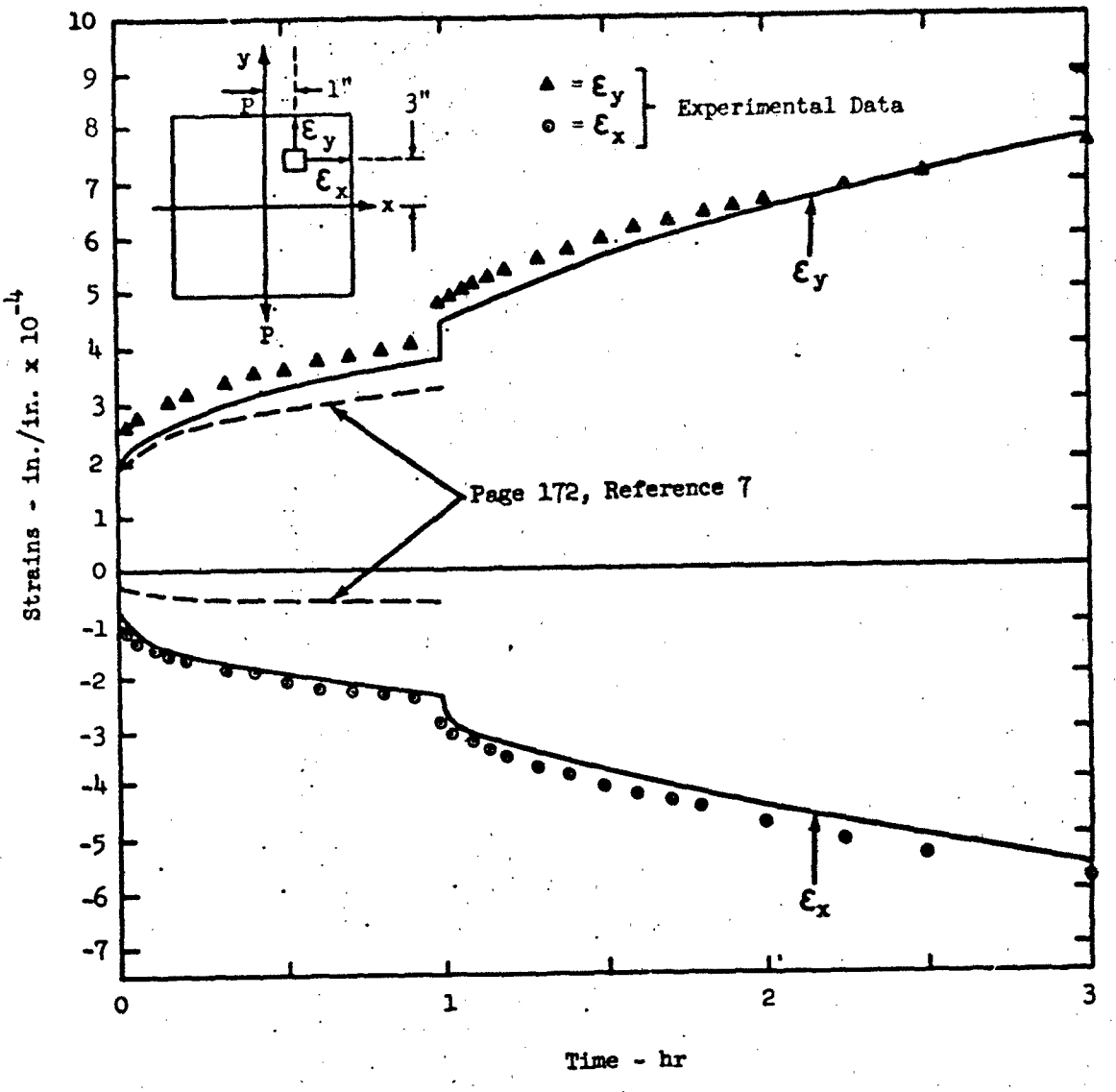


Fig. 35 Total (Elastic, Plastic and Creep) Strains at (1.0 in., 3.0 in.) (Gage 8) for P = 1600 lb to time 1 hr, then P = 2020 lb to time 3 hr

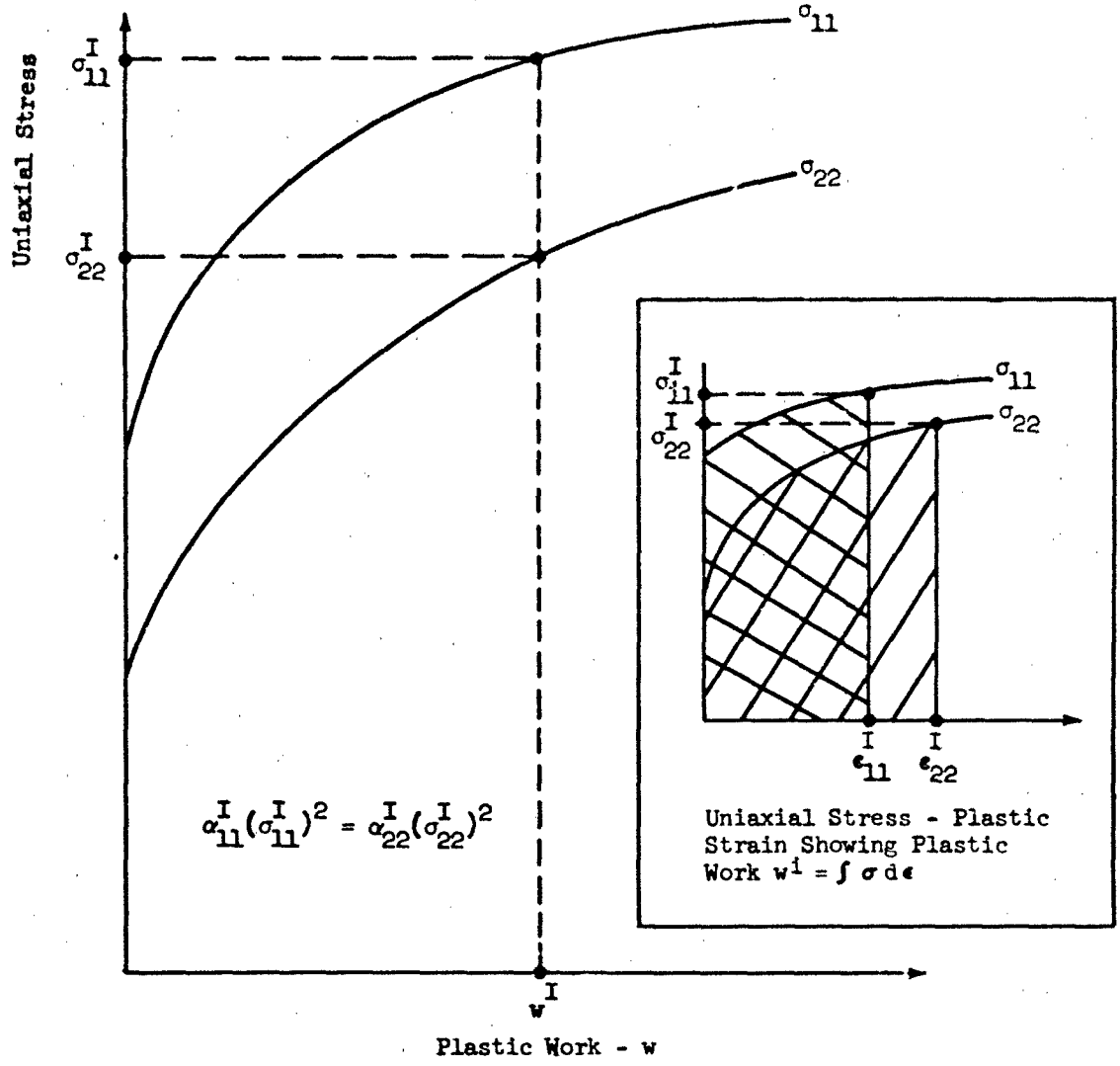


Figure 36 Typical Uniaxial Stress vs Plastic Work, w, Plot.

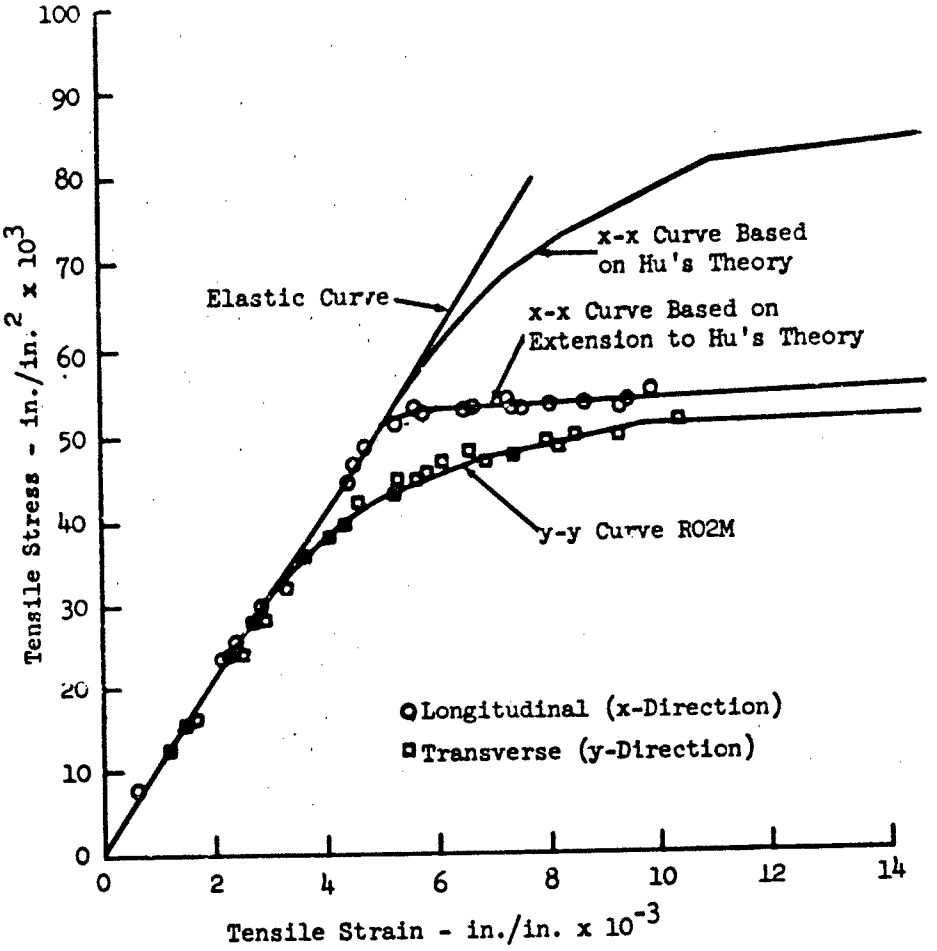


Fig. 37 Assumed Effective Stress (y-y) Curve, Calculated x-x Curves and Test Data

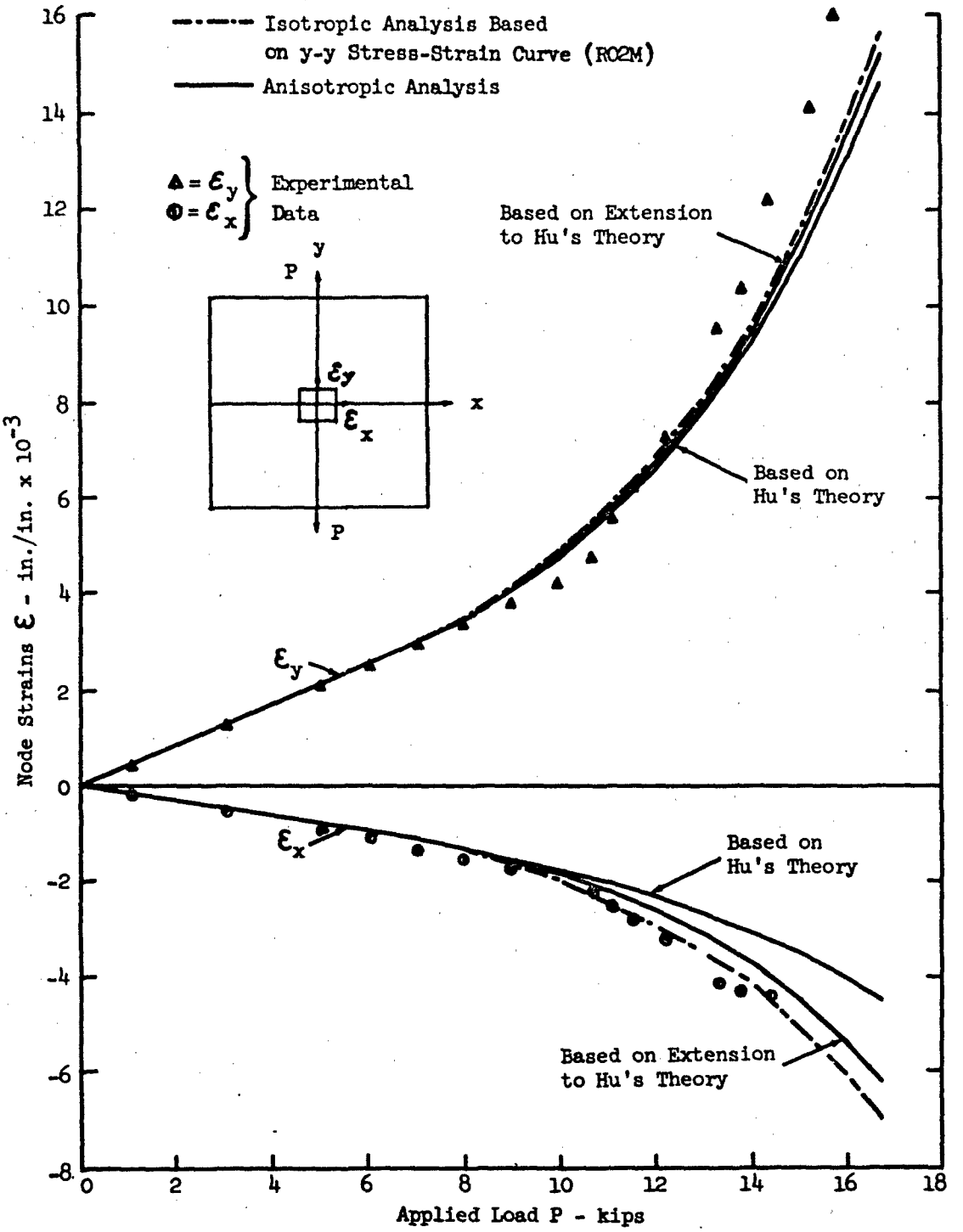


Fig. 38 Node Strains at (0, 0) - Isotropic and Anisotropic Analysis

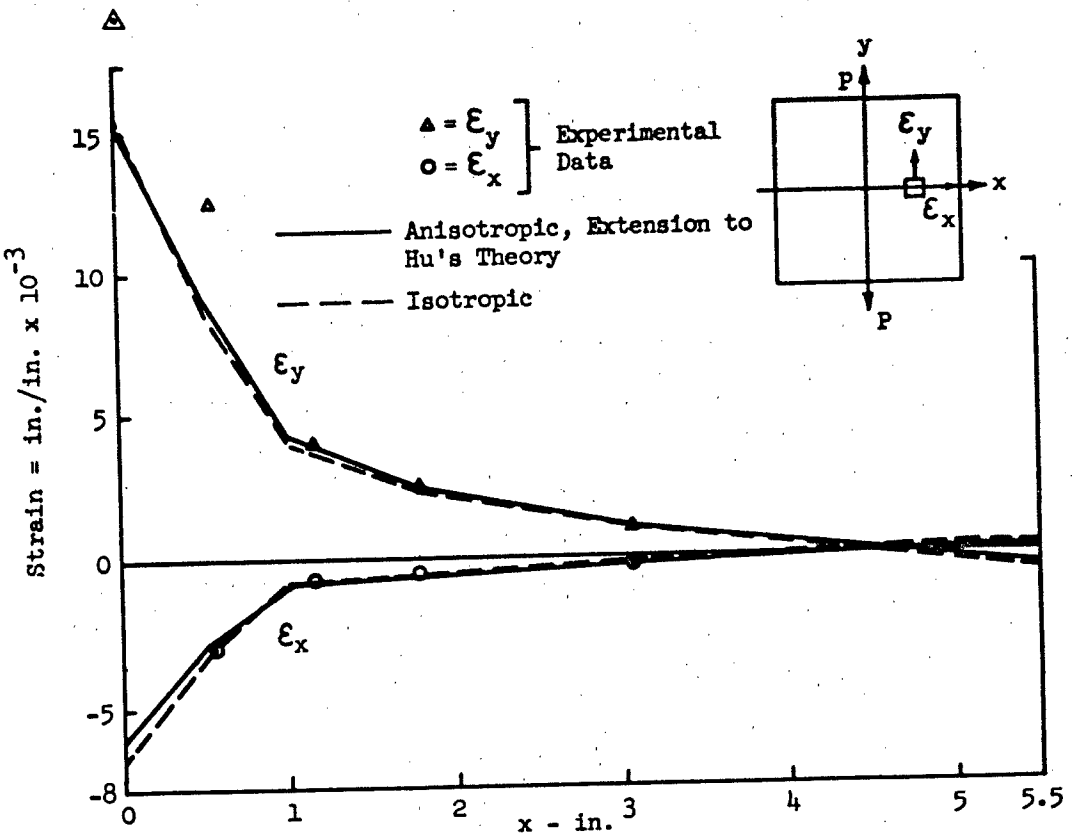


Fig. 39 Strains Along x - Axis for P = 16,760 lb.
Isotropic and Anisotropic Analyses

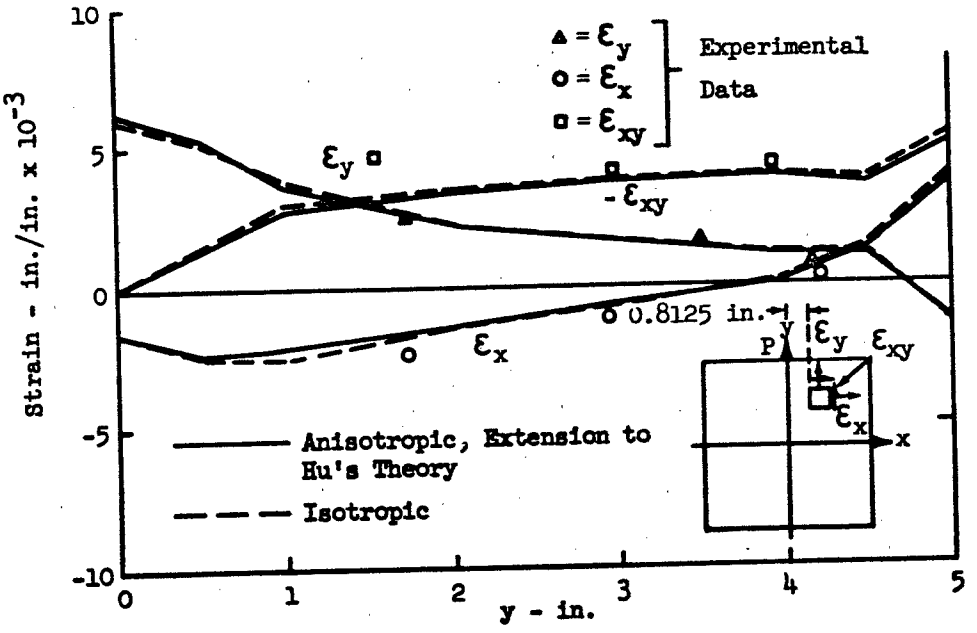


Fig. 40 Strains Along x = 0.8125 in. for P = 16,760 lb.
Isotropic and Anisotropic Analyses

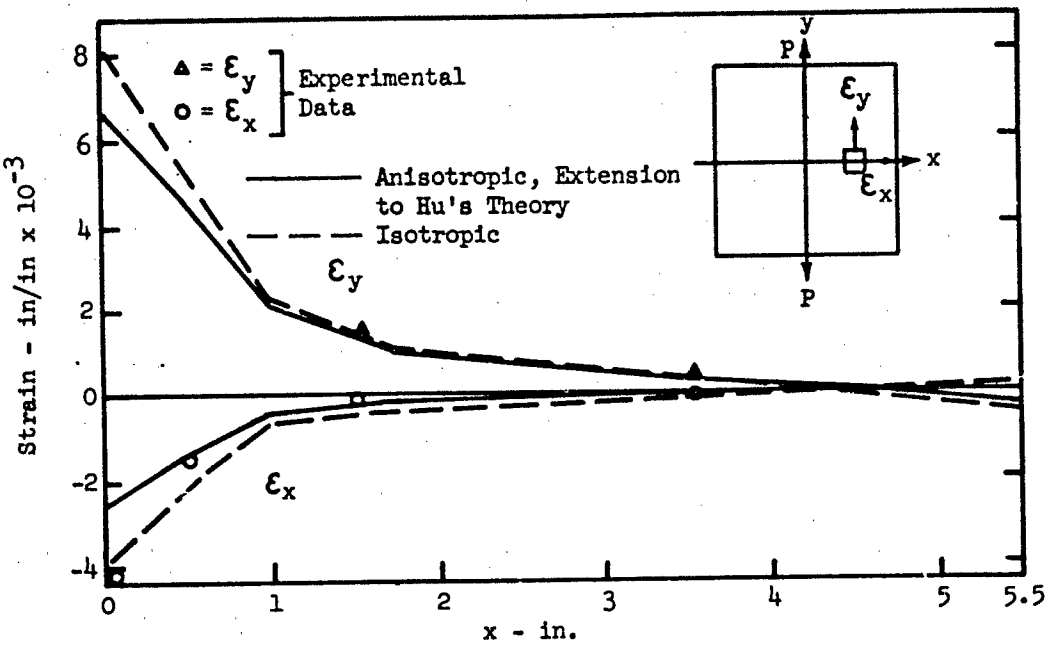


Fig. 41 Strains Along x - Axis for P = 2020 lb and t = 3.0 hr. Isotropic and Anisotropic Analyses

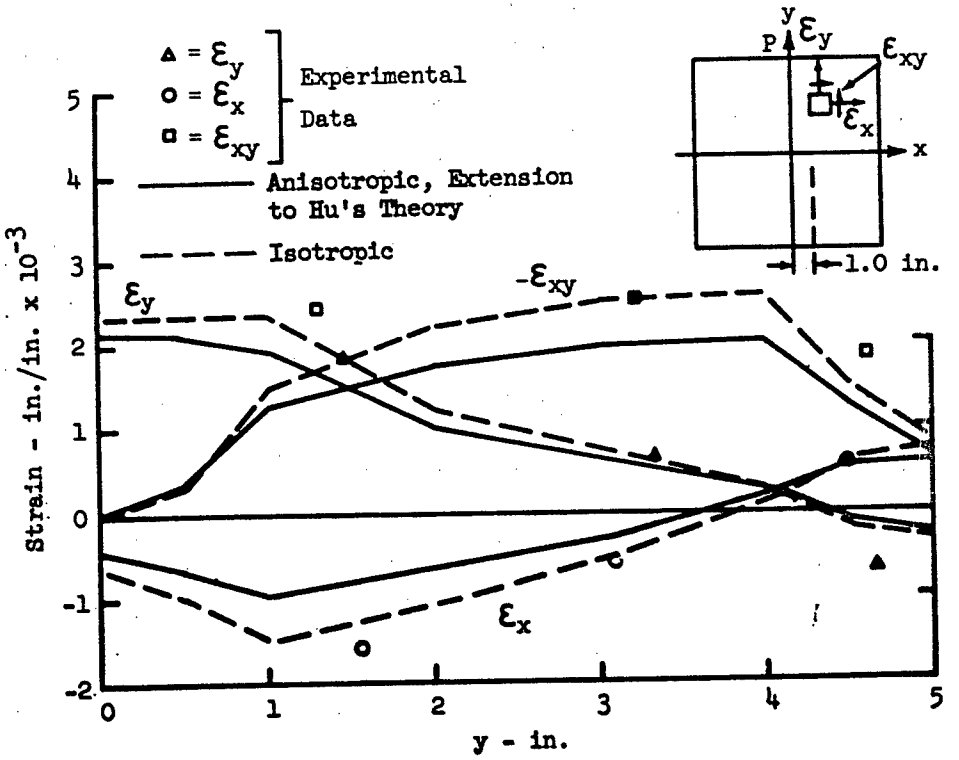


Fig. 42 Strains Along x = 1 in. for P = 2020 lb and t = 3.0 hr. Isotropic and Anisotropic Analyses

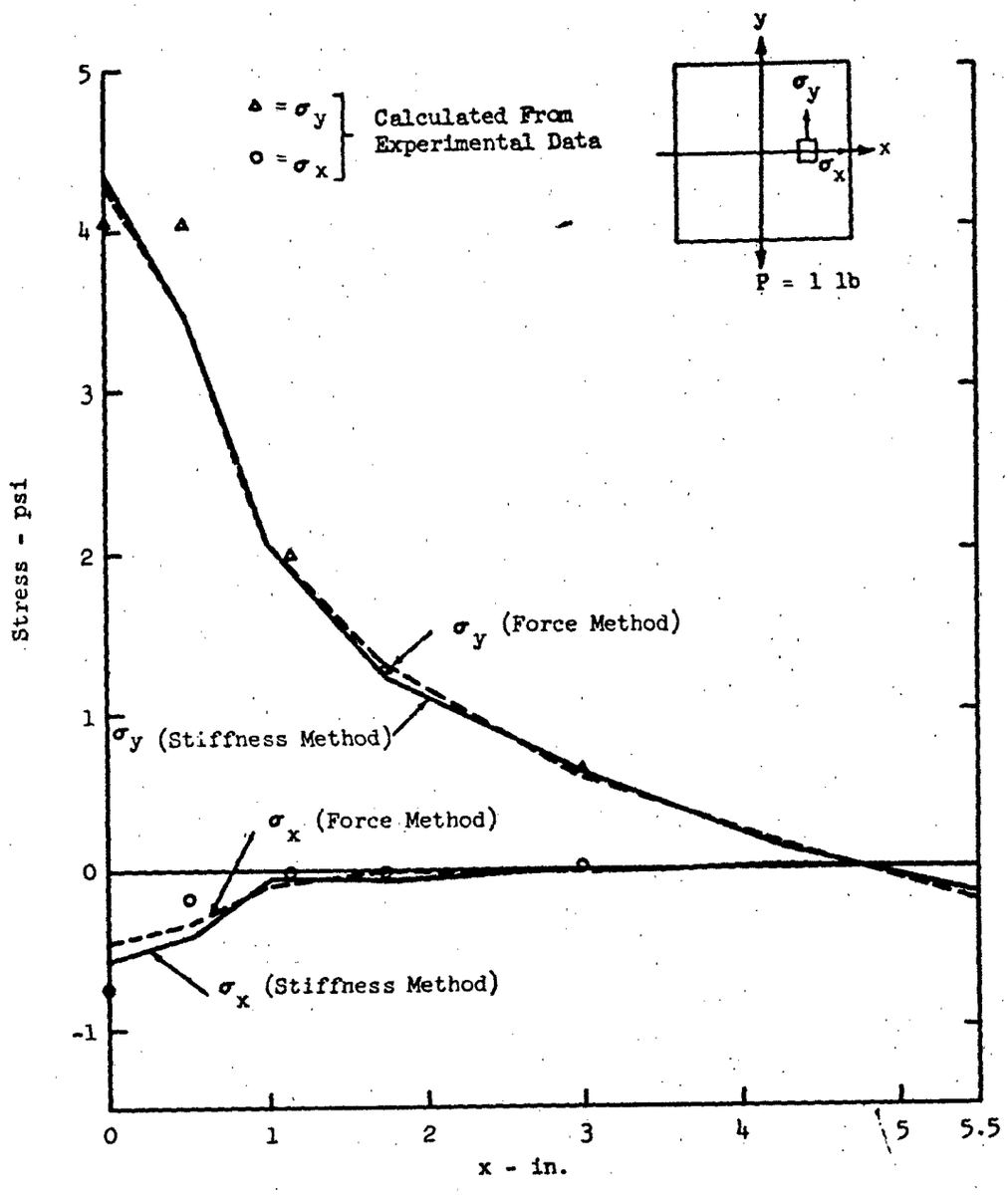


Fig. 43 Elastic Stress Distribution Along x-Axis

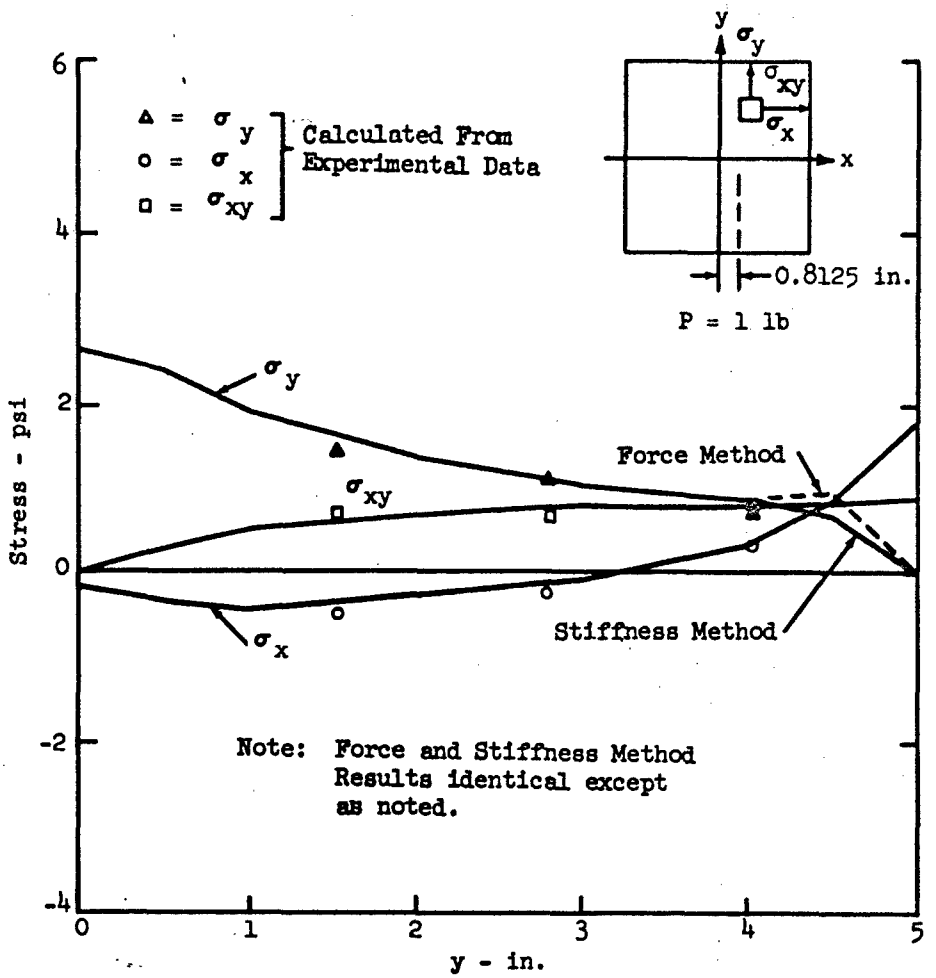


Fig. 44 Elastic Stress Distribution Along x = 0.8125 in.

APPENDIX I

ELASTIC ANALYSIS OF SHEAR LAG STRUCTURE

A. Idealization of Shear Lag Structure

As stated in the introduction, when a problem is formulated by means of a standard influence coefficient approach, the necessary linear analysis may be carried out using either the force or displacement method. Since published correlations between results of the matrix force and direct stiffness methods of linear elastic analyses for redundant structures have, in the past, left room for doubt as to the equivalence of results, this Appendix presents a comparison of the stresses from two idealizations of the simple shear-lag stiffened-plate structure, Figures (7) and (8). In the past, discrepancies have been due in part to a marked difference in the arrangement of node points for corresponding idealizations, and also to the fact that techniques for obtaining node stresses in finite element analyses are still being improved. An attempt was made to keep the idealizations as comparable as possible with respect to location of nodes and the determination of stresses.

B. Force Method

The idealization for the force method may be seen in Figure (8). It comprises conventional bars and shear panels located in the manner shown. The analyses of some previous idealizations of this type have omitted the Poisson's ratio effect. This effect can be incorporated in the manner described in Appendix II.

C. Stiffness Method

The idealization for the stiffness method consists of "Turner triangles," which are located as shown in Figure (7). The basic theory of the triangle is to be found in References 8 and 9.

While the conventional procedure was used to obtain node stresses for the force method, comparable stiffness-method stresses can be calculated in several ways. A recent paper, Reference 9, suggests two means of obtaining node stress, one of which was employed in the analysis. This method will be reviewed briefly.

Figure (10) represents a cluster of triangles. It is required to find the stresses at the node common to triangles P to W. The node forces for each triangle at this apex are obtained as described in Turner's former papers. Summing the forces on a vertical section through 1 in both directions gives

$$X_v = X_P + X_Q + X_R + X_S$$

$$Y_v = Y_P + Y_Q + Y_R + Y_S$$

Analogous results X_h and Y_h are obtained for the corresponding horizontal cut.

The node stresses at the node are

$$\sigma_x = \frac{X_v}{\frac{1}{2}(ct_p + dt_s)}$$

$$\sigma_y = \frac{Y_h}{\frac{1}{2}(at_r + bt_u}$$

$$\sigma_{xy} = \frac{1}{2}\left[\frac{X_h}{\frac{1}{2}(at_r + bt_u)} + \frac{Y_v}{\frac{1}{2}(ct_p + dt_s)}\right]$$

where a, b, c, and d are as indicated and t is the thickness.

D. Comparison of Elastic Results and Perspective

The results are correlated by means of the curves appearing in Figures (43) and (44) with calculated values from the experimental data of Reference 7.

The correlation between the stresses derived from the force method and from the stiffness method of analysis is excellent and may be regarded as exact for engineering applications. The largest discrepancy is in the direct stresses in the x direction at the middle of the plate as shown in Figure (43). Even in this region the difference is quite small. It is believed that an even closer agreement could be obtained by modifying the idealized structure to provide square shear panels adjacent to the reinforcement and a finer grid at the plate center.

The largest discrepancy between analysis and test results is located in the region of the plate center. Reference 7 indicated that considerable bending was exhibited by the structure as the ends of the stiffener were loaded. The extent to which this affects the gage readings was not determined; however, it may be anticipated that the effect be greatest near the middle of the plate. The curves on Figure (43) reinforce this impression.

On the basis of the excellent agreement noted here, it can be concluded that the incorporation of plastic and creep effects into the present method of structural analysis will not be restricted in any way by the particular linear analysis method employed.

APPENDIX II

POISSON'S RATIO EFFECT

The strain energy relationship for an elastic plate in terms of in plane stresses is given by the volume integral:

$$U = \frac{1}{2E}\int_V \left[\sigma_{xx}^2 + \sigma_{yy}^2 - 2\nu\,\sigma_{xx}\sigma_{yy} + 2(1+\nu)\sigma_{xy}^2\right]dV$$

The idealized structure corresponding to a rectangular plate for a finite element matrix force analysis has orthogonal bars taking only normal stresses and panels in pure shear. Figure (8), the shear-lag specimen, represents a typical idealization of this type. Defining $\bar{\sigma}_{xx}$ and $\bar{\sigma}_{yy}$ as the axial stress in the bars and $\bar{\sigma}_{xy}$ as the shear stress in the panels, the strain energy U' of the idealized structure is sometimes taken as:

$$U' = \frac{1}{2E}\int_V \left[\bar{\sigma}_{xx}^2 + \bar{\sigma}_{yy}^2 + 2(1+\nu)\bar{\sigma}_{xy}^2\right]dV$$

Comparing the plate strain energy U with the idealized structure strain energy U' it is obvious that the finite element expression neglects to account for the interaction term of the plate $(-2\nu\,\sigma_{xx}\sigma_{yy})$ which is due to Poisson's ratio. This uncoupling of the normal stresses has the effect of making the idealized plate less rigid than the actual plate. The finite element idealization is refined by including the term $(-2\nu\,\bar{\sigma}_{xx}\bar{\sigma}_{yy})$ in U', making the model more consistent with the elastic plate.

The sketch, Figure (9), shows a shear panel with adjacent axial load carrying bars. Assume the structure represents a portion of a plate structure ℓ inches long, b inches wide and t inches thick. The normal stresses at one corner of the idealized structure are designated σ_1 and σ_2 in the x and y directions respectively. It is sufficiently accurate in accounting for the interaction term to assume that the normal stresses are constant over the plate corresponding to the shaded quadrant and also to assume that these stresses are equal to σ_1 and σ_2, the values at the corner. The strain energy term to be included is represented by

$$\frac{1}{2}\,\sigma_1\sigma_2\,^2\,\alpha_{12}$$

where α_{12} represents the interaction flexibility influence coefficient. Carrying out the integration over the quadrant of the plate, the influence coefficient is evaluated.

$$\alpha_{12} = -\frac{1}{E}\,\nu\int_V dV = -\frac{\nu}{E}\,\frac{b\ell t}{4}$$

This term together with the reciprocal term α_{21} and similar terms for other biaxially stressed areas, when included in the flexibility matrix, account for the Poisson's ratio effect.

APPENDIX III

INSTABILITY ANALYSIS OF THE CONSTANT STRESS AND CONSTANT STRAIN METHODS

The stability of the two methods is easily tested by determining whether perturbations introduced into the analysis grow or decay with succeeding steps. It is instructive, however, to take the following approach. In place of the computed quantities, introduce exact quantities, signified by a caret and associated error terms, in the manner

$$\left\{q_i^{(k)}\right\} = \left\{\hat{q}_i^{(k)}\right\} + \left\{\xi_i^{(k)}\right\} . \qquad \text{III-1}$$

The exact relation is taken to be

$$\left\{\hat{q}_i^{(k)}\right\} = \left[\Gamma_{im}\right]\left\{P_m^{(k)}\right\} + \left[\Gamma_{ij}\right]\left\{\hat{\epsilon}_j^{(k)}\right\} . \qquad \text{III-2}$$

Equation 3 can be written in terms of member loads

$$\left\{q_i^{(k)}\right\} = \left[\Gamma_{im}\right]\left\{P_m^{(k)}\right\} + \left[\Gamma_{ij}\right]\left\{\epsilon_j^{(k-1)}\right\} \qquad \text{III-3}$$

Substituting Equation III-1 into Equation III-3 gives

$$\left\{\hat{q}_i^{(k)}\right\} + \left\{\xi_i^{(k)}\right\} = \left[\Gamma_{im}\right]\left\{P_m^{(k)}\right\} + \left[\Gamma_{ij}\right]\left\{\epsilon_j^{(k-1)}\right\}$$

from which Equation III-2 may be subtracted to yield the following expression for the errors

$$\left\{\xi_i^{(k)}\right\} = \left[\Gamma_{ij}\right]\left\{\epsilon_j^{(k-1)} - \hat{\epsilon}_j^{(k)}\right\} \qquad \text{III-4}$$

The constant stress and constant strain methods are now distinguished by the manner in which the $\epsilon_j^{(k-1)}$ are specified. The Ramberg-Osgood stress-strain relation, which we may write in the form

$$e_i^{(k)} = \frac{q_i^{(k)}}{E_i A_i} + \frac{3}{7}\frac{\sigma_{oi}}{E_i}\left|\frac{q_i^{(k)}}{\sigma_{oi} A_i}\right|^n \frac{q_i^{(k)}}{\left|q_i^{(k)}\right|}$$

will be used in examining both methods.

The nonlinear strains, in the constant stress method, are given by

$$\epsilon_j^{(k-1)} = \frac{3}{7}\frac{\sigma_{oj}}{E_j}\left|\frac{q_j^{(k-1)}}{\sigma_{oj}A_j}\right|^n \frac{q_j^{(k-1)}}{\left|q_j^{(k-1)}\right|}$$

which, on using Equation III-1, may be written in the expanded form

$$\epsilon_j^{(k-1)} = \frac{3\sigma_{oj}\hat{q}_j^{(k-1)}}{7E_j\left|\hat{q}_j^{(k-1)}\right|}\left[\left|\frac{\hat{q}_j^{(k-1)}}{\sigma_{oj}A_j}\right|^n + n\left|\frac{\hat{q}_j^{(k-1)}}{\sigma_{oj}A_j}\right|^{n-1}\frac{\xi_j^{(k-1)}}{\sigma_{oj}A_j} + \text{-----}\right]$$

A similar expansion can be constructed for the $\hat{\epsilon}_1^{(k)}$ on introducing

$$\left\{\hat{q}_j^{(k)}\right\} = \left\{\hat{q}_j^{(k-1)}\right\} + \left\{\Delta\hat{q}_j^{(k)}\right\}$$

which gives

$$\hat{\epsilon}_j^{(k)} = \frac{3\sigma_{oj}}{7E_j}\frac{\hat{q}_j^{(k)}}{\left|\hat{q}_j^{(k)}\right|}\left[\left|\frac{\hat{q}_j^{(k-1)}}{\sigma_{oj}A_j}\right|^n + n\left|\frac{\hat{q}_j^{(k-1)}}{\sigma_{oj}A_j}\right|^{n-1}\frac{\Delta\hat{q}_j^{(k)}}{\sigma_{oj}A_j} + \text{----}\right]$$

Substituting the foregoing expression into Equation III-4, and assuming small errors in the sense that $\xi_1 \ll q_1$ and small steps in the loading such that $\Delta q_1 \ll q_1$, so that terms containing the products of these quantities may be dropped, we obtain

$$\left\{\xi_i^{(k)}\right\} = \left[\Gamma_{ij}\right]\left[\frac{3n}{7E_jA_j}\frac{\hat{q}_j^{(k-1)}}{\left|\hat{q}_j^{(k-1)}\right|}\left|\frac{\hat{q}_j^{(k-1)}}{\sigma_{oj}A_j}\right|^{n-1}\right]\left\{\xi_j^{(k-1)} - \Delta\hat{q}_j^{(k)}\right\}$$

which may be re-written

$$\left\{\xi_i^{(k)}\right\} = \left[\Gamma_{ij}\right]\left[1/\bar{E}_j^{(k-1)}\ A_j\right]\left[1 - \frac{\Delta\hat{q}_i^{(k)}}{\xi_j^{(k-1)}}\right]\left\{\xi_j^{(k-1)}\right\} \qquad \text{III-5}$$

where $\bar{E}_j$ give the slope of the stress versus inelastic-strain curve at the individual element stress levels.

Instability develops when the errors in the k^{th} step increase over those in the $k\text{-}1^{st}$ step. Clearly, the method would be expected to be unstable, if any of the inequalities $\xi_i^{(k)} > \xi_i^{(k-1)}$ were satisfied directly. A more critical check, however, is the consider the entire set of $\{\xi_i\}$ as a vector (in n-dimensional space where n is equal to the number of elements) and apply the condition that the Eucliudean length of this vector does not increase. Mathematically, this means that for stability

$$\left\{\xi_i^{(k)}\right\}^T \left\{\xi_i^{(k)}\right\} \leq \left\{\xi_i^{(k-1)}\right\}^T \left\{\xi_i^{(k-1)}\right\}$$

where the critical condition is defined by using the equality sign.

For simplicity, we consider the case of infinitesimally small steps in the loading, so that the Δq_i terms may be neglected, and further, denote

$$\left[\Gamma_{ij}\right]\left[1 / \bar{E}_j^{(k-1)} \; A_j\right] = \left[B_{ij}^{(k-1)}\right]$$

The critical condition for instability then becomes

$$\left\{\xi_i^{(k)}\right\}^T \left\{\xi_i^{(k)}\right\} = \left\{\xi^{(k-1)}\right\}^T \left[B_{ij}^{(k-1)}\right]^T \left[B_{ij}^{(k-1)}\right] \left\{\xi_i^{(k-1)}\right\} \qquad \text{III-6}$$

$$= \left\{\xi_i^{(k-1)}\right\}^T \left\{\xi_i^{(k-1)}\right\}$$

It is now observed that the eigenvalues of

$$\left[B_{ij}^{(k-1)}\right]^T \left[B_{ij}^{(k-1)}\right]$$

will all be positive, hence the condition that none of these eigenvalues be greater than one (which becomes the stability requirement), can be replaced by the more severe condition that the sum of the eigenvalues not be greater than one. This latter condition can be assured by requiring that the sum of the squares of all elements of B_{ij} not be greater than one, and in addition, that the absolute sum of any row or

column in B_{ij} not be greater than one. The critical stress is then found from the largest of these sums.

In the case of the example truss problem, where $E_i = E = 10^7$ psi, $n = 10$, $A_i = 1.0$ sq. in., and $\sigma_{oi} = \sigma_o = 10^5$ psi, the sum of the squares approach, viz.,

$$\left\{\frac{3n}{7E}\left(\frac{\sigma_{cr}}{\sigma_o}\right)^{n-1}\right\}^2 \times \sum_{ij=1}^{3} \Gamma_{ij}^{\,2} = 1$$

yields $\sigma_{cr} = 83{,}800$ psi., while the rows and columns approach gives $\sigma_{cr} = 82{,}900$ psi.

Recall that the foregoing development has determined the minimum value at which instability might occur. It is of interest to compute a critical value of stress at which instability is strongly assured to occur. This can be done by returning to the initial notion that instability will occur if $\varepsilon_i^{(k)} > \varepsilon_i^{(k-1)}$. This amounts to restricting attention to the diagonal elements in the B_{ij} matrix The corresponding critical stress (lowest value) will be given by the largest (in absolute value) element on the diagonal in the B_{ij} matrix. This corresponds to the -0.414×10^7 term in the Γ_{ij} matrix, so that the critical stress is given by

$$\frac{3n}{7E}\left(\frac{\sigma_{cr}}{\sigma_o}\right)^{n-1} \times (0.414 \times 10^7) = 1$$

which gives $\sigma_{cr} = 93{,}800$ psi. The lowest stress at which instability would develop, in the case of the example truss problem, would therefore be expected to occur between 82,900 psi and 93,800 psi.

Note that if only one structural element were inelastic, then the diagonal term in the Γ_{ij} matrix corresponding to this element would give the correct critical stress by the latter procedure. Both of the foregoing values of critical stress have been indicated in Figure 4, where they are seen to correlate with the experimental (computer) results. The simple approach of considering only the diagonal elements

appears to be advantageous in the present problem. With this approach, it is easy to see that finite values of Δq_i, which make the elements in the second diagonal matrix in Equation III-5 less than one, must raise the stress for instability, which explains the progression of critical stresses (with increasing load step size) appearing in Figure 4.

Finally, it is noted that certain of the foregoing results, viz., the value of σ_{cr} based on one diagonal elements alone, can be obtained simply by introducing $q_i^{(k)} + \psi_i^{(k)}$ in place of $q_i^{(k)}$ directly into Equation III-3, and regarding the $\psi_i^{(k)}$ as perturbations on the $q_i^{(k)}$. The additional results, such as the demonstration of the effects of finite Δq_i, however, are not obtained by this procedure.

In the case of the constant strain method, in addition to the error quantities in the member loads

$$\left\{q_i^{(k)}\right\} = \left\{\hat{q}_i^{(k)}\right\} + \left\{\xi_i^{(k)}\right\}$$

we also introduce error quantities for the relaxed loads

$$\left\{q_{*i}^{(k)}\right\} = \left\{\hat{q}_{*i}^{(k)}\right\} + \left\{\xi_{*i}^{(k)}\right\} .$$

The equation defining the relaxed loads, written for the generic i^{th} member, is

$$\frac{q_i^{(k)}}{E_i A_i} + e_i^{(k-1)} = \frac{q_{*i}^{(k)}}{E_i A_i} + \frac{3q_{*i}^{(k)}}{7E_i A_i}\left|\frac{q_{*i}^{(k)}}{\sigma_{oi} A_i}\right|^{n-1}$$

which may be written in the following form

$$\left|\frac{\hat{q}_i^{(k)} + \xi_i^{(k)}}{E_i A_i}\right| \frac{q_i^{(k)}}{\left|q_i^{(k)}\right|} + \frac{3\sigma_{oi}}{7E_i}\left|\frac{\hat{q}_{*i}^{(k-1)} + \xi_{*i}^{(k-1)}}{\sigma_{oi} A_i}\right|^{n} \frac{q_{*i}^{(k-1)}}{\left|q_{*i}^{(k-1)}\right|}$$

$$= \left|\frac{q_{*i}^{(k)} + \xi_{*i}^{(k)}}{E_i A_i}\right| \frac{q_{*i}^{(k)}}{\left|q_{*i}^{(k)}\right|} + \frac{3\sigma_{oi}}{7E_i}\left|\frac{\hat{q}_{*i}^{(k-1)} + \Delta\hat{q}_{*i}^{(k-1)} + \xi_{*i}^{(k)}}{\sigma_{oi} A_i}\right|^{n} \frac{q_{*i}^{(k)}}{\left|q_{*i}^{(k)}\right|}$$

If we now apply the condition of no load reversals, then the terms $q_i^{(k)}/\left|q_i^{(k)}\right|$ etc., will all produce the same sign (for a given element) and hence may be cancelled out. Applying the condition that all error terms ξ_i and load increments Δq_i are much smaller than the load magnitude $\left|q_i\right|$, leads to the result

$$\xi_i^{(k)} = \xi_{*i}^{(k)} - \frac{E_i}{\hat{E}_{*i}^{(k-1)}} \left(\xi_{*i}^{(k-1)} - \xi_{*i}^{(k)} - \Delta q_{*i}^{(k)} \right)$$

where

$$1/\hat{E}_{*i}^{(k-1)} = \frac{3n}{7E_i} \left| \frac{\hat{q}_{*i}^{(k-1)}}{\sigma_{oi} A_i} \right|^{n-1}$$

The corresponding form of Equation III-4 may now be written by introducing the exact load-reduction-increments $\Delta\hat{q}_{rj}^{(k)}$, where

$$\Delta\hat{q}_{rj}^{(k)} = \hat{q}_j^{(k)} - \hat{q}_{*j}^{(k)}$$

which leads to

$$\left\{\xi_i^{(k)}\right\} = \left[\Gamma_{ij}\right] \left\{ \frac{3\sigma_{oj}}{7E_j} \left| \frac{\hat{q}_{*j}^{(k-1)} + \xi_{*j}^{(k-1)}}{\sigma_{oj} A_j} \right|^n \frac{q_{*j}^{(k-1)}}{\left|q_{*j}^{(k-1)}\right|} - \frac{3\sigma_{oj}}{7E_j} \left| \frac{\hat{q}_{*j}^{(k-1)} + \Delta\hat{q}_{rj}^{(k-1)} + \Delta\hat{q}_j^{(k)}}{\sigma_{oj} A_j} \right|^n \frac{\hat{q}_j^{(k)}}{\left|\hat{q}_j^{(k)}\right|} \right\}$$

Applying, once again, the smallness requirement on the ξ_i and Δq_i, yields

$$\left[1 + \frac{E_i}{\hat{E}_{*i}^{(k-1)}}\right] \left\{\xi_{*i}^{(k)}\right\} = \left[\left[\Gamma_{ij}\right] \left[1/(\hat{E}_{*j}^{(k-1)} A_j)\right] \left[1 - \frac{\Delta\hat{q}_{rj}^{(k-1)} + \Delta q_j^{(k)}}{\xi_{*i}^{(k-1)}}\right]\right] \qquad \text{III-7}$$

$$+\left[\frac{E_i}{\overline{E}^{(k-1)}_{*i}}\right]\left[1 - \frac{\Delta q_i^{(k)}}{\varsigma^{(k-1)}_{*i}}\right]\left\{\varsigma^{(k-1)}_{*i}\right\}$$

where the absolute value signs have been omitted for simplicity.

The check for the occurrence of instability may now be carried out in the same manner as for the constant stress method. Thus, considering only the case of infinitesimal load increments, the corresponding form of the B_{ij} matrix, as defined by Equation III-6, is found to be

$$\left[B_{ij}\right] = \left[\frac{\overline{E}_{(cr)i}}{E_{(cr)i} + E_i}\right]\left\{\left[\Gamma_{ij}\right]\left[1/(\overline{E}_{(cr)j}\, A_j)\right] + \left[E_i/\overline{E}_{(cr)i}\right]\right\}$$

A numerical check for the special case of the truss problem shows that the critical condition of the eigenvalues summing to unity calls for physically inadmissible values of $\overline{E}_{(cr)}$. A simpler demonstration of this property is provided by the "direct approach" (i.e., setting $\varsigma_i^{(k)} = \varsigma_{*i}^{(k-1)}$). In this case, the critical stress is given by the lowest value corresponding to the "n" equations obtained by equating the diagonal elements of

$$\left[1 + \frac{E_i}{E_{(cr)i}}\right]^{-1}\left\{\left[\Gamma_{ij}\right]\left[1/(\overline{E}_{(cr)j}\, A_j)\right] + \left[E_i/\overline{E}_{(cr)i}\right]\right\} = \left[\pm\ 1\right] .$$

In the example truss problem, where $E_i = E$, $\sigma_{oi} = \sigma_o$, and $A_i = 1.0$, and where the three diagonal elements in the Γ_{ij} matrix can be denoted by $-\zeta_i E$, where in turn $0<\zeta_i<1$, the foregoing matrix equation reduces to the following simple algebraic equation

$$\frac{-\zeta_i E + E}{\overline{E}_{(cr)i} + E} = \pm 1 .$$

The indicated critical values are easily seen to be $-\zeta_i E$ and $-(2 - \zeta_i)E$, both of which are physically inadmissible for the Ramberg-Osgood stress-strain relation. Thus, the constant strain method is indicated to be free of instability in this case. The problem of accuracy, of course, is another matter, due to the necessity of working with finite (and preferably large) load steps. These, apparently, are responsible for the slow divergence of the computer results shown in Figure 5.

APPENDIX IV

ROTATION OF AXES OF ANISOTROPY

Let the x and y axes be rotated from the axes of anisotropy in a positive sense so that from the strain transformation equations we get:

$$\left.\begin{aligned} d\epsilon_x &= \ell^2 d\epsilon_{11} + m^2 d\epsilon_{22} + 2\ell m d\epsilon_{12} \\ d\epsilon_y &= m^2 d\epsilon_{11} + \ell^2 d\epsilon_{22} - 2\ell m d\epsilon_{12} \\ d\epsilon_{xy} &= -\ell m d\epsilon_{11} + \ell m d\epsilon_{22} + 2(\ell^2 - m^2) d\epsilon_{12} \end{aligned}\right\} \text{IV-1}$$

when ℓ, and m are the usual direction cosines of the x-axis with respect to the orthogonal axes.

Similarly from the stress transformation equations:

$$\left.\begin{aligned} \sigma_{11} &= \ell^2 \sigma_x + m^2 \sigma_y - 2\ell m \sigma_{xy} \\ \sigma_{22} &= m^2 \sigma_x + \ell^2 \sigma_y + 2\ell m \sigma_{xy} \\ \sigma_{12} &= \ell m \sigma_x - \ell m \sigma_y + (\ell^2 - m^2)\alpha_{xy} \end{aligned}\right\} \qquad \text{IV-2}$$

Substituting Equation IV-2 into the appropriate expressions in Equation 12 gives a set of equations as follows:

$$\left.\begin{aligned} d\epsilon_{11} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[(\ell^2\alpha_{11} - m^2\alpha_{12})\sigma_x + (m^2\alpha_{11} - \ell^2\alpha_{12})\sigma_y \right. \\ &\quad \left. - 2\ell m(\alpha_{11} + \alpha_{12})\sigma_{xy}\right] \\ d\epsilon_{22} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\left[(m^2\alpha_{22} - \ell^2\alpha_{12})\sigma_x + (\ell^2\alpha_{22} - m^2\alpha_{12})\sigma_y \right. \\ &\quad \left. + 2\ell m(\alpha_{22} + \alpha_{12})\sigma_{xy}\right] \\ d\epsilon_{12} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\, 3\alpha_{44}\left[\ell m(\sigma_x - \sigma_y) + (\ell^2 - m^2)\sigma_{xy}\right] \end{aligned}\right\} \text{IV-3}$$

Finally substituting Equation IV-3 into Equation IV-1, simplifying and defining three new coefficients δ, δ_1 and δ_2 such that $\delta = \alpha_{12} - \frac{3}{2}\alpha_{44}$, $\delta_1 = (\alpha_{11} + \delta)$ and $\delta_2 = (\alpha_{22} + \delta)$ the appropriate incremental flow equations become:

$$\left.\begin{aligned}
d\epsilon_x &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\Big[(\ell^4\delta_1 + m^4\delta_2 - \delta)\sigma_x - (\alpha_{12} - \ell^2m^2\delta_1 - \ell^2m^2\delta_2)\sigma_y \\
&\quad - 2\ell m(\ell^2\delta_1 - m^2\delta_2)\sigma_{xy}\Big] \\
d\epsilon_y &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\Big[(\ell^4\delta_2 + m^4\delta_1 - \delta)\sigma_y - (\alpha_{12} - \ell^2m^2\delta_2 - \ell^2m^2\delta_1)\sigma_x \\
&\quad - 2\ell m(m^2\delta_1 - \ell^2\delta_2)\sigma_{xy}\Big] \\
d\epsilon_{xy} &= \frac{d\bar{\epsilon}}{\bar{\sigma}}\Big[(3\alpha_{44} + 4\ell^2m^2\delta_1 + 4\ell^2m^2\delta_2)\sigma_{xy} \\
&\quad - 2\ell m(\ell^2\delta_1 - m^2\delta_2)\sigma_x - 2\ell m(m^2\delta_1 - \ell^2\delta_2)\sigma_y\Big]
\end{aligned}\right\} \text{IV-4}$$

Insertion of Equation IV-2 into 11 gives the expression for effective stress:

$$\left.\begin{aligned}
\bar{\sigma}^2 &= \Big[(\ell^4\delta_1 + m^4\delta_2 - \delta)\sigma_x^2 - 2(\alpha_{12} - \ell^2m^2\delta_1 - \ell^2m^2\delta_2)\sigma_x\sigma_y \\
&\quad + (\ell^4\delta_2 + m^4\delta_1 - \delta)\sigma_y^2 + (3\alpha_{44} + 4\ell^2m^2\delta_1 + 4\ell^2m^2\delta_2)\sigma_{xy}^2 \\
&\quad - 4\ell m(\ell^2\delta_1 - m^2\delta_2)\sigma_x\sigma_{xy} - 4\ell m(m^2\delta_1 - \ell^2\delta_2)\sigma_y\sigma_{xy}\Big]
\end{aligned}\right\} \text{IV-5}$$

Now suppose we make an uniaxial stress-strain test for the x-direction. The valid expressions become:

$$d\epsilon_x = \frac{d\bar{\epsilon}}{\bar{\sigma}}(\ell^4\delta_1 + m^4\delta_2 - \delta)\sigma_x$$

$$\bar{\sigma}^2 = (\ell^4\delta_1 + m^4\delta_2 - \delta)\sigma_x^2$$

From the three Equations IV-4, we obtain equations similar to Equation 15. This equation is augmented under restrictions of equal plastic work thus:

$$a\sqrt{\alpha_{11}} = b\sqrt{\alpha_{22}} \quad c\sqrt{\alpha_{33}} = d\sqrt{\ell^4\delta_1 + m^4\delta_2 - \delta} \qquad \text{IV-6}$$

Assigning the value of unity to one "α" as before we obtain the relative values of the others. In particular we now obtain a value for ($l^4\delta_1 + m^4\delta_2 - \delta$) which is a function of α_{11}, α_{22}, α_{33} all known and α_{44} unknown. Therefore α_{44} may be determined.

APPENDIX V

INELASTIC MATRIX COMPUTER PROGRAM

A. Program Description

This is a brief description of the Grumman inelastic matrix program for carrying out elastic-plastic-creep analysis - deck No. 45128 - Elastic Unloading (follows Hooke's Law when unloading), including anisotropy.

This program for elastic-plastic analysis has proved to be quite adaptable for analytic investigations. It has been modified to include an option for anisotropic analysis. Previously, the program was modified to use time-hardening theory, deformation theory, inelastic unloading, and constant stress theory (none of which have been retained in the final program). It was not necessary to do major program revisions to accommodate these variations. The program housekeeping is arranged so that modifications to the manipulation of data will not affect the housekeeping. Thus the program makes a convenient framework to explore various calculation procedures.

The program is written in Fortran II to run on the Grumman IBM 7094s. These are 2 channel (A and B) machines with 6 drives per channel; 32,768 words of core storage; on-line card reader; on-line printer; and printer clock. The program is set to run under Fortran Monitor control which uses a $JOB card for identification. Input is on logical tape 7 (A-2), print output on logical tape 6 (A-2) and punched output on logical tape 5 (B-4) (not used by this program). Logical tape 8 (B-1) is used for storage of binary output which is converted to BCD print output in link 6 (at the end of job). The program will accept an input data tape on logical tape 9 (A-5) and will write a binary save tape for restart on logical tape 11 (A-6). These 2 auxiliary tapes are optional for each run (see description of the control cards). The Grumman IOU subroutine, as well as the subroutines for rewinding and unloading a tape (RUN) and for moving to the start of a designated file (FILTAP) are included, as required in the program, in column binary form.

The program tape furnished to Wright Patterson Air Force Base contains all the information needed to duplicate our analysis. It is in the following sequence:

File 1 - a 1-card BCD label tested by the program to distinguish BCD data tape from binary save tape.

File 2 - BCD card images for matrices SIM AND SIJ. See pages 79 and 80 for a description of the matrices and their format.

File 3 - BCD card images of Fortran program (6 links), each link including binary subroutines previously mentioned.

Multiple (6) end of file marks.

We recommend that all 3 files of this tape be copied for use, then that file 3 be punched onto Fortran cards. The punching program must be able to handle mixed-mode cards to accommodate the short binary subroutines. This deck of cards, with the proper *I.D. or $JOB card, will then be in proper form for a Fortran compile and execute, using the copied tape on drive A-5 (logical 9). The program uses only the data from files 1 and 2; it will not move into file 3. This 2-file format for the data tape is generated by the GISMO matrix system in use at Grumman and elsewhere.

Each link of the program contains the non-IBM subroutines needed for operation. Standard input-output subroutines etc. will be taken from the library tape. As a point of information, the program contains six links numbered consecutively from 1 to 6.

1) Link 1 reads the first control card, and reads all other decimal input supplied.

2) Link 2 is used only on a restart job. It reads the modified step table, if provided, and part of the binary input tape.

3) Link 3 is used only on a restart job, and reads the balance of the binary input tape.

4) Link 4 is the processing link. It does all the calculation, print control, and writing of binary output on tape B-1 to be converted to BCD print output by Link 6.

5) Link 5 writes a binary tape for restart, then transfers to Link 6. If no binary tape is to be written, exit is from Link 6.

6) Link 6 reads the binary output stored on tape B-1 and converts it to BCD output on tape A-3 for printing. When tape B-1 has been completely processed, a message to the operator indicates that it need not be saved. If, due to machine error or operator intervention, tape B-1 is not processed into prints on A-3, but is saved, then Link 6 can be used as a separate program using B-1 as a data tape and will process B-1 into prints on A-3. Link 6 does not use any data from COMMON. All necessary clues are stored on B-1.

Sequencing and details of the data cards follow. The symbols used in the program for various items of input data are listed on page 77 and are shown on the sample key-punching sheet page 78.

The data cards are used in the following sequence, immediately after the *DATA card required by the Monitor:

1. General clue card (FORMAT 1) containing KLU4, NINCLD, NA, KLU7, KLUISO, ALPHA, BETA, GAMMA, GNU, and a title or caption.

2. Table of load or time steps desired. Up to ten cards defining ten steps may be used. Load steps and time steps may be used in any sequence. The maximum level for each load or time step may be above or below the previous maximum level of load or time. The program verifies the algebraic sign of the increment, and corrects it if necessary. Each card contains four variables TEMP1, TEMP2, TEMP3, TEMP4 in FORMAT 2.

3. Data matrices. These may be provided in any sequence. Each matrix has a header card in FORMAT 3, one or more data cards in FORMAT 4, and a blank card to end it. The last input matrix on the Monitor input tape must be followed by one added blank card (two total) to trip the program into operation.

In the event that a job is running overtime, and it is desired to stop the program in a restartable form, mount a blank tape on logical tape 11 (drive A-6) and set Sense Switch 6 on. This will write the contents of memory on A-6 in proper form to continue the run later. The program distinguishes between the saved binary tape and a decimal input tape at starting time. Either is mounted on logical tape 9 (Drive A-5).

Built-in pauses in the program are as follows:

1. Pause 11111 to mount the data input tape at the start of the program, if all matrices are not on the Monitor input tape A-2. For this KLU4, in the first data card, should be a "1" to "4" indicating the count of decimal matrices on A-5. If A-5 is a binary saved tape from a preceding run, any digit (except zero) acceptable for KLU4.

2. Pause 1 to mount a blank tape on A-6 to receive memory. This is reached either with Sense Switch 6, or with a card in the table of steps punched TAPE in columns 7-10.

B. Symbols and Format of the Data Cards

1. General Clue Card - FORMAT 1

Cols.	Field	Symbol	
1	I1	KLU4	This gives the number of input matrices on the auxiliary input tape (A-5). If all matrices are on the monitor tape, leave this blank. If using binary restart tape, use a digit. Maximum number of decimal input matrices on the auxiliary input tape is 4.

Cols.	Field	Symbol	
2-4	I3	NINCLD	This gives the number of non-increment recycles at each load or time level. The total cycles at each level is NINCLD + 1. When printing, the first, tenth, twentieth, etc., and last cycle of each printable level will be printed.
5-7	I3	NA	This sets the frame size for the problem to be handled. NA is three times the number of nodes. Maximum value is 165 (55 nodes).
8	I1	KLU7	0, or blank, prints 5 preselected matrices on cycles indicated by the step table; 1 prints all matrices on cycles indicated by the step table. 2 prints 5 preselected matrices on all cycles; 3-9 print all matrices on all cycles.
9	I1	KLUISO	0 indicates an isotropic run 1 indicates an anisotropic run
10	1X		Not used
11-20	E10.3	ALPHA	A variable defined by the creep-strain equation.
21-30	E10.3	BETA	A variable defined by the creep-strain equation.
31-35	F5.2	GAMMA	A variable defined by the creep-strain equation.
36-40	F5.2	GNU	Poisson's ratio "nu"
41-50	10X		Not used
51-80	5A6	TA-TE	Any 30 characters of alpha-numeric text to be printed as a heading for identification purposes.

2. Table of Steps (limited to ten entries) - FORMAT 2

Cols.	Field	
1-6	6X	Not used
7-10	A4	LOAD indicates a load step, TIME indicates a time step, TAPE indicates write memory on a save-tape on drive A-6 (logical #11) then exit.

Cols.	Field	
		FINS or blank indicates end of table (this may be the 11th card in the table).
11-20	E10.6	Upper limit of step in pounds or hours
21-30	E10.1	Interval or increment for calculation
31-40	E10.1	Interval or increment for print output. Prints are generated on tape A-3 for the current cycle when the current load or time level is an integral multiple of the print interval. If the print interval is left blank, no prints are generated for cycles in this step. If print interval is very small compared to the current level, then numeric problems sometimes occur in the print control subroutine (OUTPUT), and it may be necessary to re-run with the every-cycle print control "2" or "3" for KLU7 punched in the first control card.
41-80		Ignored

3. Data Matrices - Header Card - FORMAT 3

Cols.	Field	
1-6	6X	Not used
7-10	4X	Not used. We use the letters MTRX for compatibility with the GISMO Matrix System, which reads and writes matrices in this format.
11	1X	Not used
12-17	A6	This is the identification name for the input matrices and must correspond exactly with one of the following names:

bbbSIM Matrix of stresses for applied loads maximum size 165x1

bbbSLJ Matrix of stresses for member strains maximum size 165x165

bTMIGN Table of stress values 11x1)
) These two matrices define the stress-strain curve as a series of chords. The data is entered in this format
bTEPSN Table of strain values 11x1)

merely to conform to the format of the SIM-SLJ matrices which were generated using the GISMO Matrix System. Note that the first value in both TSIGN an TEPSN must be zero to avoid upsetting the interpolation procedure.

TALF 12 α_{12}
TALF 23 α_{23}
TALF 31 α_{31}
TALF 44 α_{44}
} Basic anisotropic parameters 11x1

18	1X	Not used
19-21	I3	Number of rows in this matrix
22-24	I3	Number of columns in this matrix
25-32	8X	Not used
33	I1	The digit 3
34-80		Not used

4. Data Matrices - Data Cards - FORMAT 4

Cols.	Field	
1	1X	Not used
2-4	I3	Row index for the first element
5-7	I3	Column index for the first element
8-23	E16.8	The first element on this card
24	1X	Not used
25-27	I3	Row index for the second element
28-30	I3	Column index for the second element
31-46	E16.8	The second element on this card
47	1X	Not used
48-50	I3	Row index for the third element
51-53	I3	Column index for the third element

Cols.	Field	
54-69	E16.8	The third element on this card
70-80		Not used

The last card of a matrix must be completely blank (tested in Col. 2-4). The last matrix on the Monitor input tape 7 (drive A-2) must be followed by one added blank card (two total) to trip the program into operation.

The input matrices on the Monitor tape may be in any sequence as long as each matrix starts with a header card, has all its data cards next, and ends with a blank card.

C. Anisotropic Parameter Matrices

If a run is indicated as isotropic in the first control card (KUUISO in Col. 9 is zero or blank) the program will read in the anisotropic parameters, if provided; then it will replace them with the built-in parameters for the isotropic condition. If a run is indicated as anisotropic in the first control card (KUUISO is 1 to 9), then one each of the anisotropic parameter matrices must be provided or the program will terminate on an error.

D. Restart Procedure

If the run being set up is a restart, the input deck can be in several forms. The first control card must have a digit in KUU4 so that the program will read tape A-5; NINCLD and KUU7 are read from this card. The other factors are carried from the previous run. The continuation of an isotropic run will always be isotropic, and conversely, regardless of the clue provided (KUUISO).

The table of steps may be read in again (modified) if the previous run is stopped with sense switch 6, or it may be retained and continued from the previous run. However, if the previous run is stored on tape A-6 by using a TAPE card in the step table, then a new step table must be read in.

In either case, the last data card on a restart job must be FINS or blank in columns 7-10. This means a restart data deck will have a minimum of two cards (clue card and a blank), or a maximum of 12 cards (clue card, 10 step cards and a FINS or blank).

E. Time Estimates

For time estimates, allow 3.5 minutes to compile the Fortran, 1.5 minutes to read the input tape, 2.0 seconds per cycle printed, and 100 to 110 cycles of calculation per minute. Each printed cycle writes approximately 3 feet of print tape (55 node problems).

01	02	03 04	05 06	07 08	09 10	11 12	13 14	15 16	17 18	19 20	21 22	23 24	25 26	27 28	29 30	31 32	33 34	35 36	37 38	39 40	41
	GENERAL CONTROL CARD - ONE REQUIRED.															FORMAT I					
I1		I3		I3		I1	I1	IX		E10.3						E10.3				F5.2		F5.2		
KLU 4		NINCLD		NA		KLU 7	KLU ISO			ALPHA						BETA				GAMMA		GNU		
	STEP TABLE (ONE TO TEN CARDS PLUS A FINS OR BLANK CARD)																				
		6X		A4			E10.6					E10.1					E10.1				
				TEMP1			TEMP2					TEMP3					TEMP4				
				LØAD		UPPER LIMIT OF STEP				INCREMENT FOR CALCULATION						INCREMENT FOR PRINT					
				TIME		UPPER LIMIT OF STEP				INCREMENT FOR CALCULATION						INCREMENT FOR PRINT					
				TAPE																	
				FIN$																	
	MATRIX HEADER CARDS (DEFINE THE NAME, SIZE AND START OF A MATRIX																				
		6X		4X		IX	A6		IX	I3	I3			8X			II				
							NAME			NROWS	NCOLS						← NFORM				
				MTRX			$IM			SIZE	1						3				
				MTRX			$IJ			SIZE	SIZE						3				
				MTRX			T$IGN			11	1						3				
				MTRX			TEP$N			11	1						3				
				MTRX		TALF12				11	1						3		(TALF23, TAL		
	MATRIX DATA CARDS (THREE ELEMENTS PER CARD - ROW SORT - LAS																				
IX		I3	I3				E16.8					IX	I3	I3					E16.8		
		NR(1)	ND(1)				EL(1)						NR(2)	ND(2)					EL(2)		
		ROW	COL.				ELEMENT						ROW	COL.					ELEMENT		
	EACH MATRIX IS TERMINATED BY ONE COMPLETELY BLANK CARD																				
01	02	03 04	05 06	07 08	09 10	11 12	13 14	15 16	17 18	19 20	21 22	23 24	25 26	27 28	29 30	31 32	33 34	35 36	37 38	39 40	41

CODING INSTRUCTIONS:

1. Alphabetical characters are written as follows
   A B C D E F G H I J K L M N $ P Q R $ T U V W X Y Z
2. Numerical characters are written as follows
   1 2 3 4 5 6 7 8 9 0

otropic = Anisotropic

= Elastic Unloading (Follows Hooke's Law)

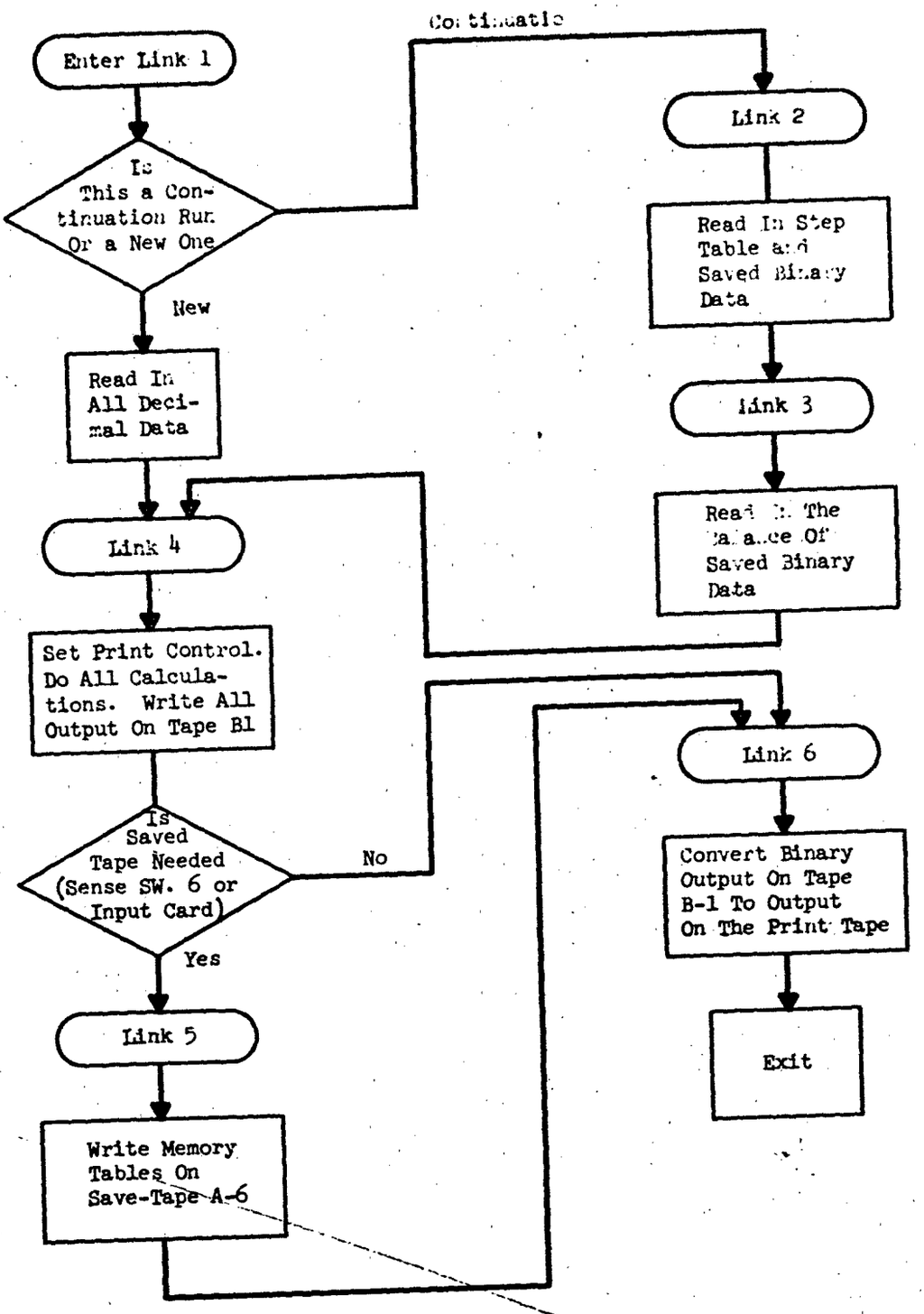
39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
							10X														5A6																					
															TA-TE						(TITLE OR HEADING)																					
)				FORMAT 2																																							
IT				VARY LOAD LEVEL AT CONSTANT TIME																																							
IT				VARY TIME, HOLD LOAD LEVEL CONSTANT																																							
				STORE MEMORY ON TAPE A-6 (LOGICAL NO. 11) FOR RESTART, THEN EXIT																																							
				END OF TABLE ENTRIES (BLANK CARD IS ALSO ACCEPTED)																																							
ATRIX)				FORMAT 3																																							
3, TALF31 AND TALF44 ARE SIMILAR)																																										
- LAST CARD MAY HAVE 1 OR 2 ELEMENTS) FORMAT 4																																									
16.8								IX	I3			I3										E16.8																			
(2)									NR(3)			ND(3)										EL(3)																			
ENT									ROW			COL										ELEMENT																			
RD																																									
39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80

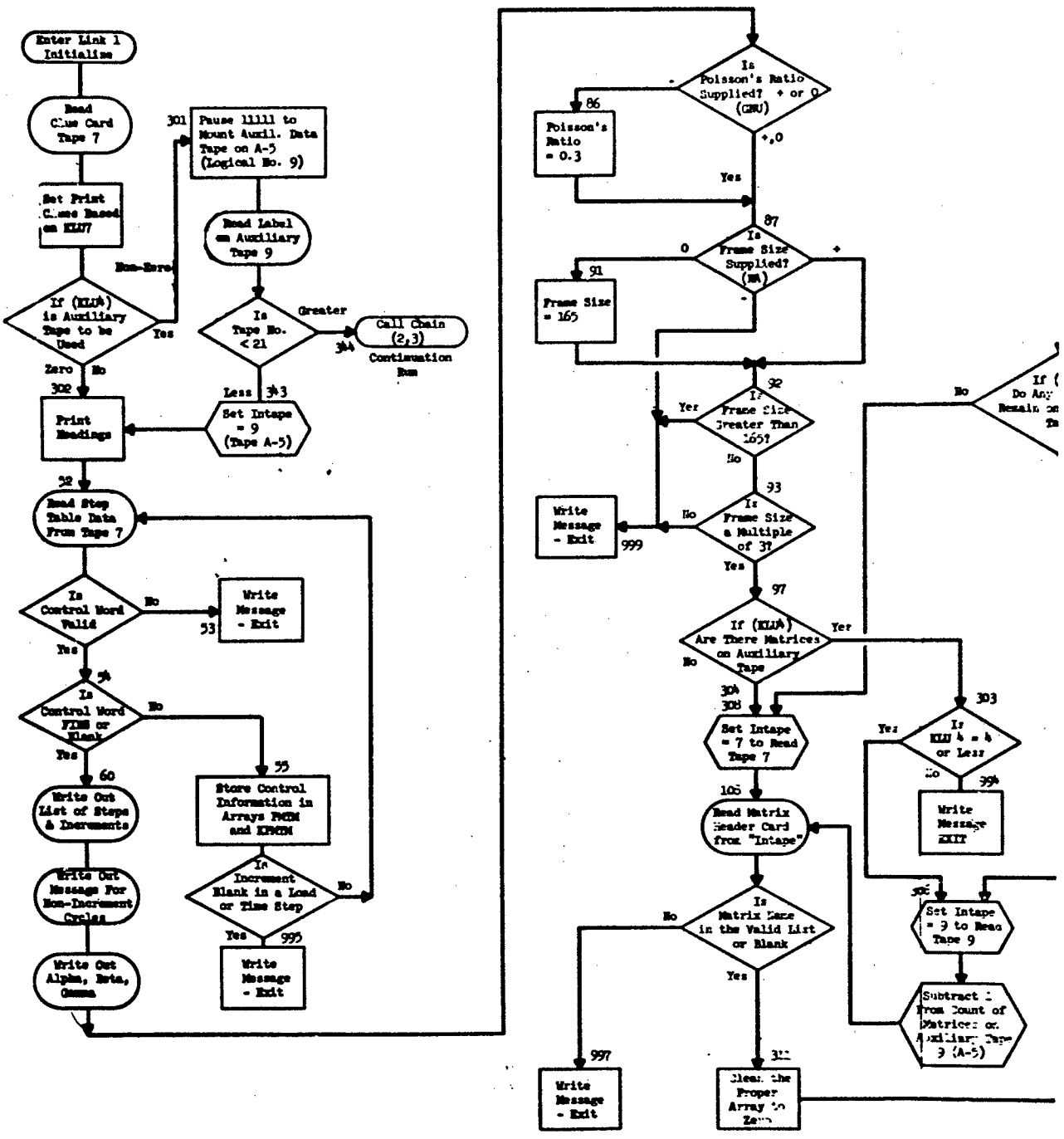
2

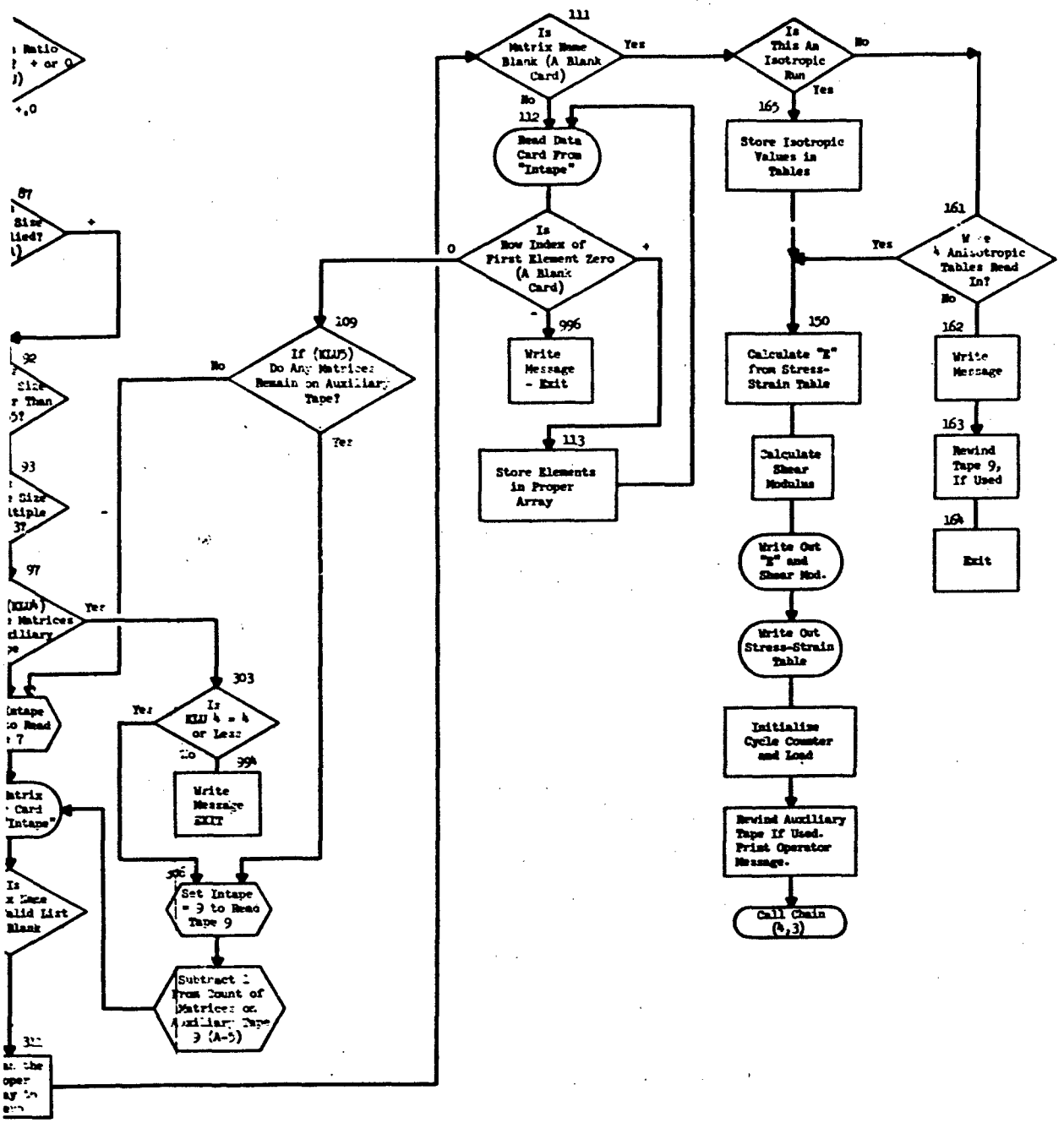
FLOW CHART FOR

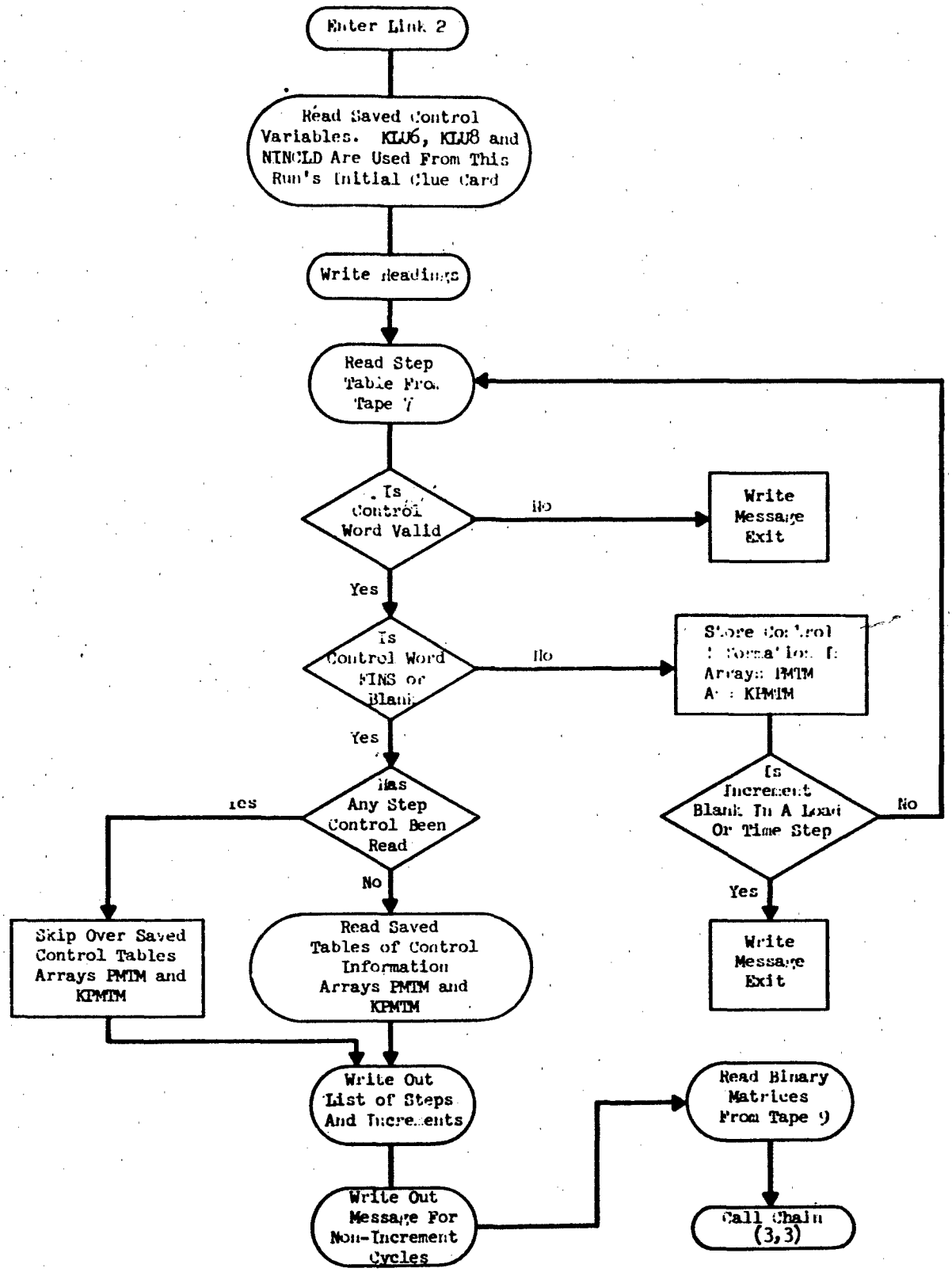
INELASTIC PLATE

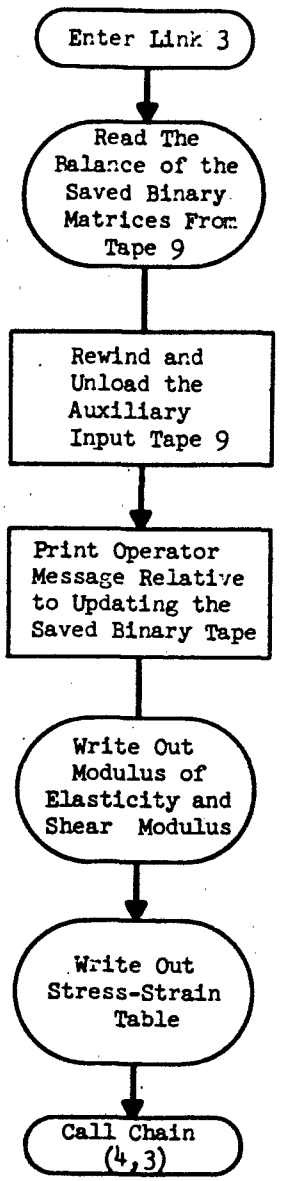
ANALYSIS

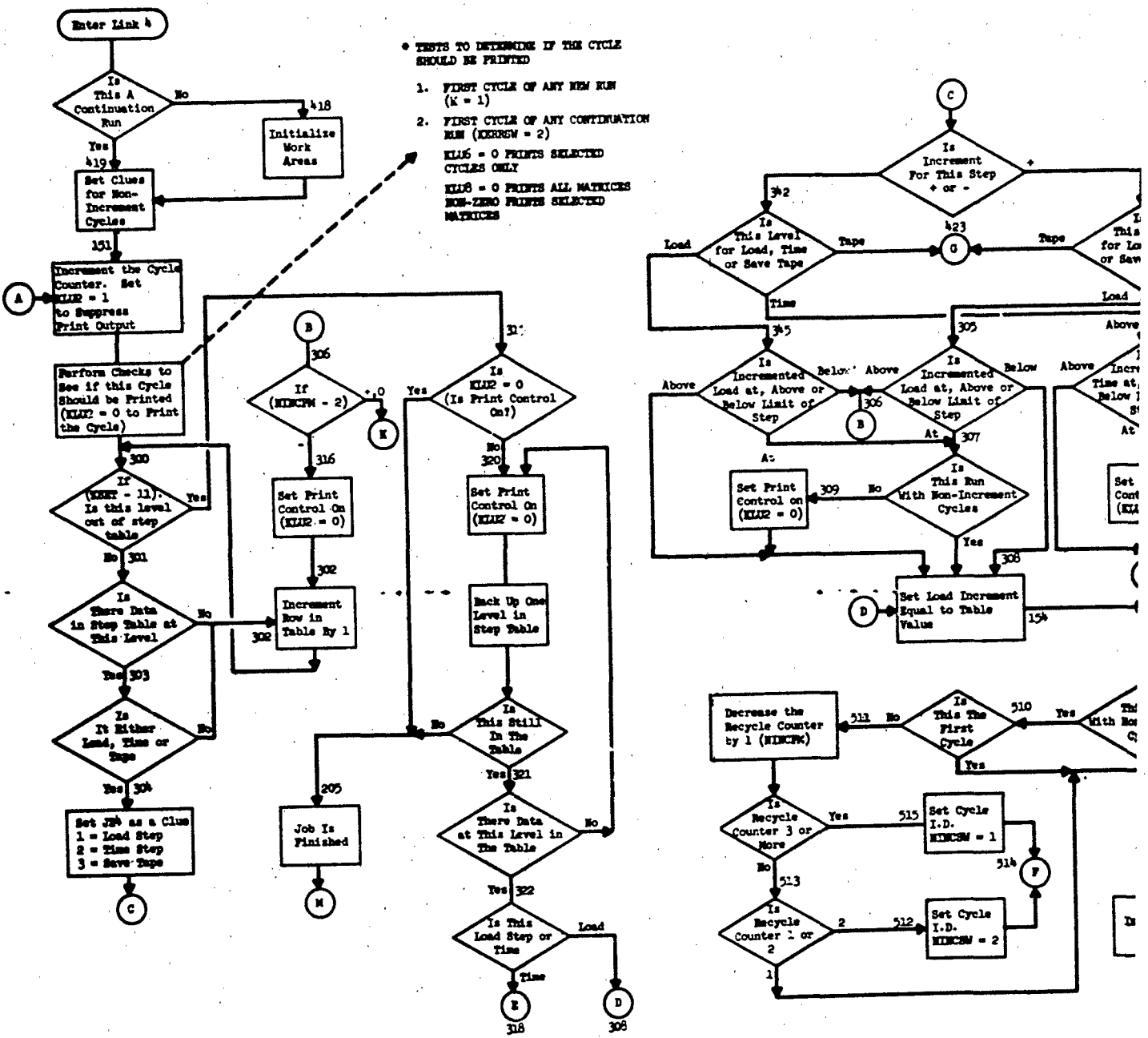

Enter Link 1
Is This a Continuation Run Or a New One
Continuatio
New
Read In All Decimal Data
Link 4
Set Print Control. Do All Calculations. Write All Output On Tape B1
Is Saved Tape Needed (Sense SW. 6 or Input Card)
No
Yes
Link 5
Write Memory Tables On Save-Tape A-6
Link 2
Read In Step Table and Saved Binary Data
Link 3
Read In The Balance Of Saved Binary Data
Link 6
Convert Binary Output On Tape B-1 To Output On The Print Tape
Exit

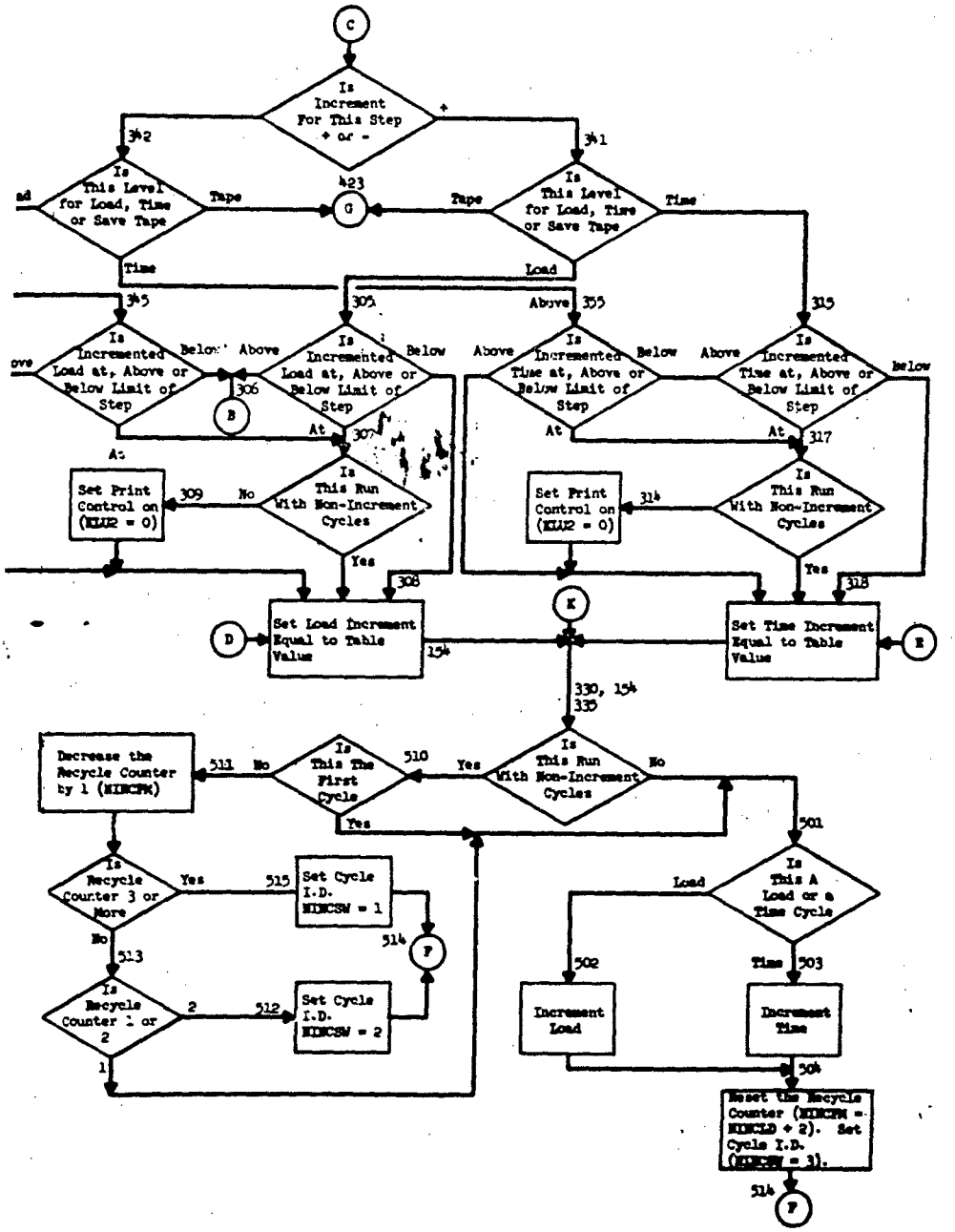

Enter Link 1 Initialize
Read Clue Card Tape 7
Set Print Clues Based on KLM7
If (KLM) is Auxiliary Tape to be Used
Non-Zero
Yes
Zero
No
302
Print Headings
301
Pause 11111 to Mount Auxil. Data Tape on A-5 (Logical No. 9)
Read Label on Auxiliary Tape 9
Is Tape No. < 21
Greater
304
Call Chain (2,3) Continuation Run
Less
303
Set Intape = 9 (Tape A-5)
52
Read Step Table Data From Tape 7
Is Control Word Valid
No
53
Write Message - Exit
Yes
54
Is Control Word FINS or Blank
No
Yes
60
Write Out List of Steps & Increments
Write Out Message For Non-Increment Cycles
Write Out Alpha, Beta, Gamma
55
Store Control Information in Arrays FMIN and KFMIN
Is Increment Blank in a Load or Time Step
No
Yes
995
Write Message - Exit
Is Poisson's Ratio Supplied? + or 0 (GNU)
-
86
Poisson's Ratio = 0.3
+,0
Yes
87
Is Frame Size Supplied? (MM)
0
+
-
91
Frame Size = 165
92
Is Frame Size Greater Than 165?
Yes
No
93
Is Frame Size a Multiple of 3?
No
Yes
999
Write Message - Exit
97
If (KLM) Are There Matrices on Auxiliary Tape
Yes
No
304
308
Set Intape = 7 to Read Tape 7
303
Is KLM 4 = 4 or Less
Yes
No
305
Write Message EXIT
306
Set Intape = 9 to Read Tape 9
Subtract 1 From Count of Matrices on Auxiliary Tape 9 (A-5)
105
Read Matrix Header Card from "Intape"
Is Matrix Name in the Valid List or Blank
No
Yes
997
Write Message - Exit
311
Clear the Proper Array to Zero

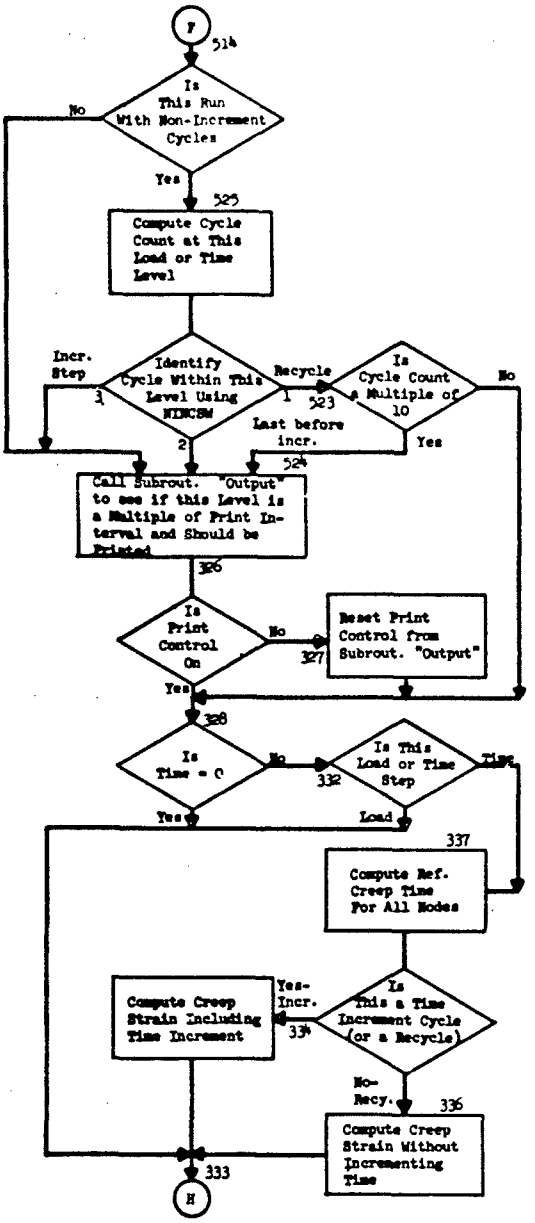

111
Is Matrix Name Blank (A Blank Card)
Yes
No
112
Read Data Card From "Intape"
Is Row Index of First Element Zero (A Blank Card)
0
+
-
996
Write Message - Exit
113
Store Elements in Proper Array
109
If (KLUS) Do Any Matrices Remain on Auxiliary Tape?
No
Yes
303
Is KLU 4 = 4 or Less
Yes
No
99A
Write Message EXIT
Set Intape = 9 to Read Tape 9
Subtract 1 From Count of Matrices on Auxiliary Tape 9 (A-5)
Is This An Isotropic Run
No
Yes
165
Store Isotropic Values in Tables
150
Calculate "E" from Stress-Strain Table
Calculate Shear Modulus
Write Out "E" and Shear Mod.
Write Out Stress-Strain Table
Initialize Cycle Counter and Load
Rewind Auxiliary Tape If Used. Print Operator Message.
Call Chain (4,3)
161
Were 4 Anisotropic Tables Read In?
Yes
No
162
Write Message
163
Rewind Tape 9, If Used
164
Exit

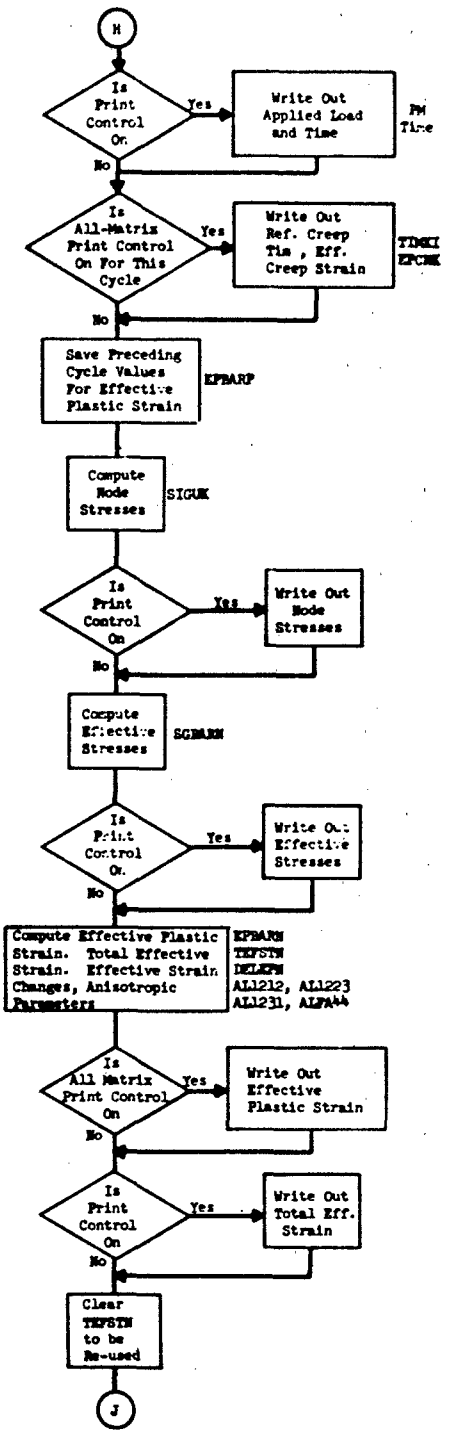

Enter Link 2
Read Saved Control Variables. KLU6, KLU8 and NINCLD Are Used From This Run's Initial Clue Card
Write Headings
Read Step Table Fro. Tape 7
Is Control Word Valid
No
Write Message Exit
Yes
Is Control Word FINS or Blank
No
Store Control Information In Arrays PMTM And KPMTM
Yes
Has Any Step Control Been Read
Yes
No
Is Increment Blank In A Load Or Time Step
No
Yes
Skip Over Saved Control Tables Arrays PMTM and KPMTM
Read Saved Tables of Control Information Arrays PMTM and KPMTM
Write Message Exit
Write Out List of Steps And Increments
Read Binary Matrices From Tape 9
Write Out Message For Non-Increment Cycles
Call Chain (3,3)

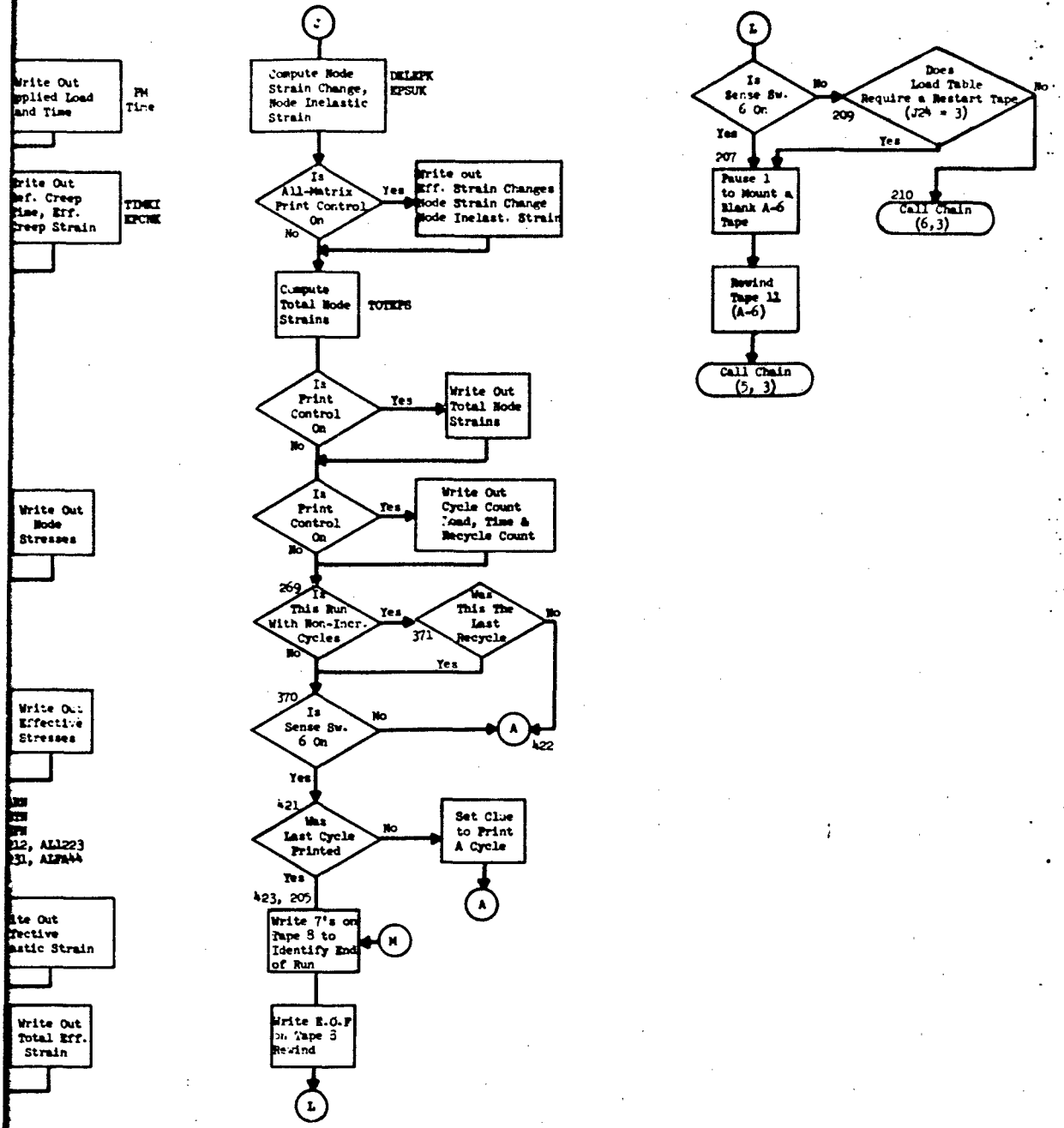

Enter Link 3
Read The Balance of the Saved Binary Matrices From Tape 9
Rewind and Unload the Auxiliary Input Tape 9
Print Operator Message Relative to Updating the Saved Binary Tape
Write Out Modulus of Elasticity and Shear Modulus
Write Out Stress-Strain Table
Call Chain (4,3)


Enter Link 4
Is This A Continuation Run
No
418
Initialize Work Areas
Yes
419
Set Clues for Non-Increment Cycles
151
Increment the Cycle Counter. Set KLU2 = 1 to Suppress Print Output
A
Perform Checks to See if this Cycle Should be Printed (KLU2 = 0 to Print the Cycle)
300
If (KERT - 11). Is this level out of step table
Yes
No
301
Is There Data in Step Table at This Level
No
302
Increment Row in Table By 1
Yes
303
Is It Either Load, Time or Tape
No
Yes
304
Set JSB as a Clue 1 = Load Step 2 = Time Step 3 = Save Tape
C
B
306
If (NINCRW - 2)
<0
K
>0
316
Set Print Control On (KLU2 = 0)
302
205
Job Is Finished
M
Is KLU2 = 0 (Is Print Control On?)
Yes
No
320
Set Print Control On (KLU2 = 0)
Back Up One Level in Step Table
Is This Still In The Table
No
Yes
321
Is There Data at This Level in The Table
No
Yes
322
Is This Load Step or Time
Load
Time
E
318
D
308
• TESTS TO DETERMINE IF THE CYCLE SHOULD BE PRINTED
1. FIRST CYCLE OF ANY NEW RUN (K = 1)
2. FIRST CYCLE OF ANY CONTINUATION RUN (KERRSW = 2)
KLU6 = 0 PRINTS SELECTED CYCLES ONLY
KLU8 = 0 PRINTS ALL MATRICES NON-ZERO PRINTS SELECTED MATRICES
C
Is Increment For This Step + or -
+
342
Is This Level for Load, Time or Save Tape
Load
Tape
Time
423
Tape
345
305
Is Incremented Load at, Above or Below Limit of Step
Above
Below
At
306
B
Is Incremented Load at, Above or Below Limit of Step
Below
Above
At
307
Set Print Control on (KLU2 = 0)
309
No
Is This Run With Non-Increment Cycles
Yes
308
D
Set Load Increment Equal to Table Value
154
Decrease the Recycle Counter by 1 (NINCRW)
511
No
Is This The First Cycle
510
Yes
Yes
Is Recycle Counter 3 or More
Yes
515
Set Cycle I.D. NINCSW = 1
514
F
No
513
Is Recycle Counter 2 or 2
2
512
Set Cycle I.D. NINCSW = 2
1

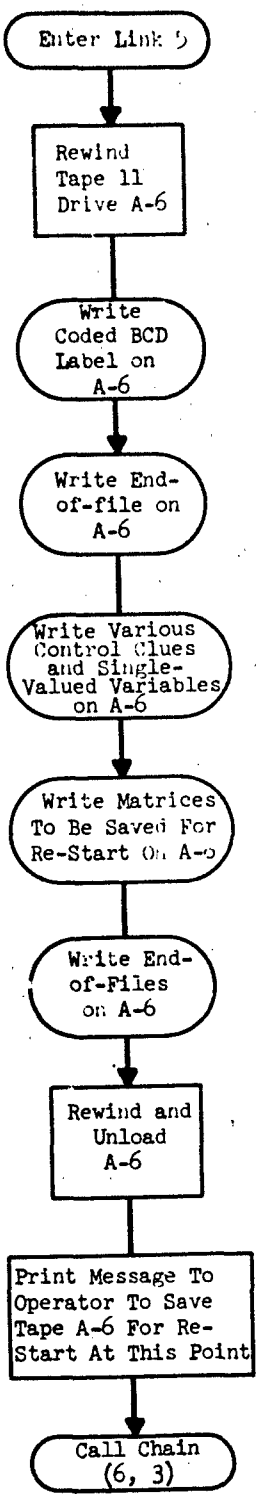

C
Is Increment For This Step + or -
+
342
341
Is This Level for Load, Time or Save Tape
Tape
423
G
Tape
Is This Level for Load, Time or Save Tape
Time
Time
Load
345
305
Above
355
315
Is Incremented Load at, Above or Below Limit of Step
Below
Above
Is Incremented Load at, Above or Below Limit of Step
Below
Above
Is Incremented Time at, Above or Below Limit of Step
Below
Above
Is Incremented Time at, Above or Below Limit of Step
Below
306
B
At
307
At
At
317
Set Print Control on (KLU2 = 0)
309
No
Is This Run With Non-Increment Cycles
Set Print Control on (KLU2 = 0)
314
Is This Run With Non-Increment Cycles
Yes
Yes
308
318
D
Set Load Increment Equal to Table Value
154
K
Set Time Increment Equal to Table Value
E
330, 154
335
Decrease the Recycle Counter by 1 (NIRCPM)
511
No
Is This The First Cycle
510
Yes
Is This Run With Non-Increment Cycles
No
Yes
501
Is Recycle Counter 3 or More
Yes
515
Set Cycle I.D. NIRCSW = 1
514
F
No
513
Is Recycle Counter 1 or 2
2
512
Set Cycle I.D. NIRCSW = 2
1
Load
Is This A Load or a Time Cycle
502
Time
503
Increment Load
Increment Time
504
Reset the Recycle Counter (NIRCPM = NIRCLD + 2). Set Cycle I.D. (NIRCSW = 1).
514
F

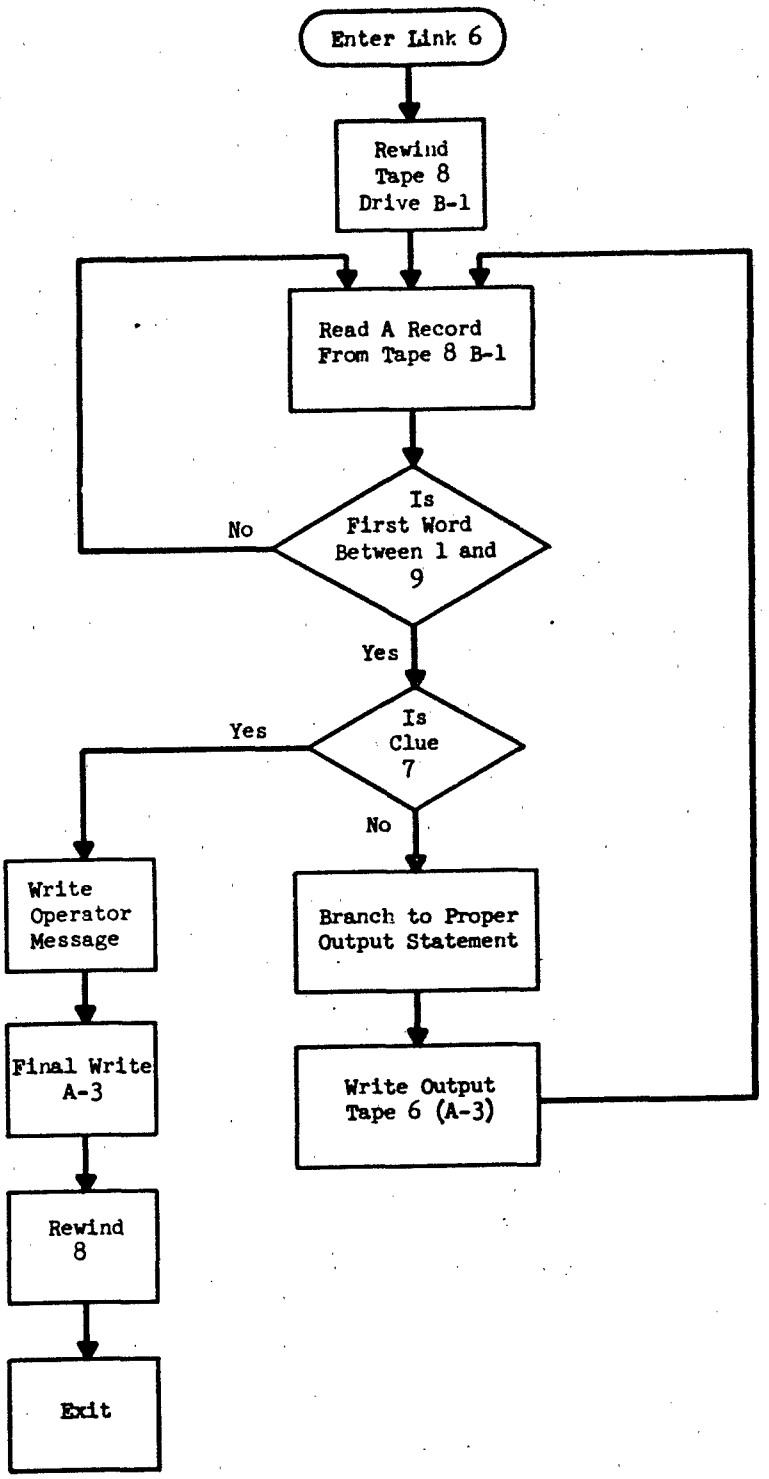

F
524
Is This Run With Non-Increment Cycles
No
Yes
525
Compute Cycle Count at This Load or Time Level
Identify Cycle Within This Level Using NINCSW
Incr. Step
1
Recycle
1
523
Is Cycle Count a Multiple of 10
No
Yes
2
Last before incr.
524
Call Subrout. "Output" to see if this Level is a Multiple of Print Interval and Should be Printed
326
Is Print Control On
No
327
Reset Print Control from Subrout. "Output"
Yes
328
Is Time = 0
No
332
Is This Load or Time Step
Time
Load
Yes
337
Compute Ref. Creep Time For All Nodes
Is This a Time Increment Cycle (or a Recycle)
Yes-Incr.
334
Compute Creep Strain Including Time Increment
No-Recy.
336
Compute Creep Strain Without Incrementing Time
333
H


H
Is Print Control On
Yes
Write Out Applied Load and Time
PM
Time
No
Is All-Matrix Print Control On For This Cycle
Yes
Write Out Ref. Creep Tim , Eff. Creep Strain
TIMKI
EPCHK
No
Save Preceding Cycle Values For Effective Plastic Strain
EPBARP
Compute Node Stresses
SIGUK
Is Print Control On
Yes
Write Out Node Stresses
No
Compute Effective Stresses
SGBARN
Is Print Control On
Yes
Write Out Effective Stresses
No
Compute Effective Plastic Strain. Total Effective Strain. Effective Strain Changes, Anisotropic Parameters
EPBARN
TEPSTN
DELEPN
AL1212, AL1223
AL1231, ALFA44
Is All Matrix Print Control On
Yes
Write Out Effective Plastic Strain
No
Is Print Control On
Yes
Write Out Total Eff. Strain
No
Clear TEPSTN to be Re-used
J


Write Out Applied Load and Time
PM Time
Write Out Ref. Creep Time, Eff. Creep Strain
TDMCK EPCMK
Write Out Mode Stresses
Write Out Effective Stresses
212, AL1223
231, AL1244
Write Out Effective Plastic Strain
Write Out Total Eff. Strain
Compute Mode Strain Change, Mode Inelastic Strain
DELEPK EPSUK
Is All-Matrix Print Control On
Yes
No
Write out Eff. Strain Changes Mode Strain Change Mode Inelast. Strain
Compute Total Mode Strains
TOTEPS
Is Print Control On
Write Out Total Mode Strains
Is Print Control On
Write Out Cycle Count Load, Time & Recycle Count
269
Is This Run With Non-Incr. Cycles
371
Was This The Last Recycle
Is Sense Sw. 6 On
370
A
422
Was Last Cycle Printed
421
Set Clue to Print A Cycle
423, 205
Write 7's on Tape 8 to Identify End of Run
M
Write E.O.F on Tape 8 Rewind
L
Is Sense Sw. 6 On
209
Does Load Table Require a Restart Tape (J24 = 3)
207
Pause 1 to Mount a Blank A-6 Tape
210
Call Chain (6,3)
Rewind Tape 12 (A-6)
Call Chain (5, 3)

2 8


Enter Link 5
Rewind Tape 11 Drive A-6
Write Coded BCD Label on A-6
Write End-of-file on A-6
Write Various Control Clues and Single-Valued Variables on A-6
Write Matrices To Be Saved For Re-Start On A-6
Write End-of-Files on A-6
Rewind and Unload A-6
Print Message To Operator To Save Tape A-6 For Re-Start At This Point
Call Chain (6, 3)


Enter Link 6
Rewind Tape 8 Drive B-1
Read A Record From Tape 8 B-1
Is First Word Between 1 and 9
No
Yes
Is Clue 7
Yes
No
Write Operator Message
Final Write A-3
Rewind 8
Exit
Branch to Proper Output Statement
Write Output Tape 6 (A-3)

FORTRAN II LISTING

FOR INELASTIC PLATE

ANALYSIS

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*     XEQ                                                                0001
*     CHAIN(1,3)                                                         0002
*     LIST8                                                              0003
 451281  GRUMMAN AIRCRAFT ENGINEERING CORP.   DECK NO. 45128   LINK 1

C     MATRIX ANALYSIS FOR ISOTROPIC OR ANISOTROPIC INELASTIC STRUCTURES   0005
C     IN THE PLASTIC AND CREEP REGIME                                    0006
C                                                                        0007
C                                                                        0008
C     THIS PROGRAM WRITTEN FOR AIR FORCE CONTRACT AF 33(615)-2260        0009
C                                                                        0010
C     * * *              TABLE OF SYMBOLS USED              * * *       0011
C                                                                        0012
C          * * ARRAYS * *                                                0013
C     AL1212 - MATRIX OF ANISOTROPIC PARAMETERS FOR EACH NODE            0014
C     AL1223 - MATRIX OF ANISOTROPIC PARAMETERS FOR EACH NODE            0015
C     AL1231 - MATRIX OF ANISOTROPIC PARAMETERS FOR EACH NODE            0016
C     ALFA44 - MATRIX OF ANISOTROPIC PARAMETERS FOR EACH NODE            0017
C     DELEPK - MATRIX OF NODE STRAIN CHANGES (DELTA EPSILON)             0018
C              FOR THE X AND Y DIRECTIONS, AND THE SHEAR STRAIN CHANGE    0019
C     DELEPN - MATRIX OF EFFECTIVE INELASTIC STRAIN CHANGES              0020
C     EPBARN - MATRIX OF EFFECTIVE INELASTIC STRAINS (EPSILON BAR SUB N) 0021
C              FOR K-TH CYCLE                                            0022
C     EPBARP - MATRIX OF EFFECTIVE INELASTIC STRAINS (EPSILON BAR SUB N) 0023
C              FOR K-1 CYCLE (THE PRECEDING CYCLE)                       0024
C     EPCNK  - MATRIX OF EFFECTIVE CREEP STRAINS FOR THIS CYCLE          0025
C     EPCNP  - MATRIX OF EFFECTIVE CREEP STRAINS FOR PREVIOUS CYCLE      0026
C     EPSUK  - MATRIX OF NODE PLASTIC STRAINS (EPSILON SUB U) FOR THE    0027
C              X, Y AND SHEAR STRAINS, FOR THE K-TH CYCLE                0028
C     KPMTM  - TABLE OF CONTROL CLUES (LOAD STEP, TIME STEP, WRITE SAVE  0029
C              TAPE)  READ-IN CONTROL INFORMATION USED WITH PMTM         0030
C     PMTM   - TABLE OF LOAD INCREMENTS, PRINT CONTROL INCREMENTS AND    0031
C              UPPER LOAD LEVEL PER STEP (CONTROL INFORMATION)           0032
C     SGBARM - MATRIX OF EFFECTIVE NODE STRESSES FOR LAST CYCLE THAT     0033
C              SHOWED AN INCREASE AT A PARTICULAR NODE                   0034
C     SGBARN - MATRIX OF EFFECTIVE NODE STRESSES FOR CURRENT CYCLE       0035
C     SGBARP - MATRIX OF EFFECTIVE NODE STRESSES FOR PREVIOUS CYCLE      0036
C     SIGUK  - MATRIX OF NODE STRESSES (SIGMA SUB U) FOR K-TH CYCLE      0037
C     SIJ    - MATRIX OF STRESSES FOR MEMBER STRAINS                     0038
C     SIM    - MATRIX OF STRESSES FOR APPLIED LOADS                      0039
C     TALF12 - MATRIX OF ANISOTROPIC PARAMETERS (INPUT DATA)             0040
C     TALF23 - MATRIX OF ANISOTROPIC PARAMETERS (INPUT DATA)             0041
C     TALF31 - MATRIX OF ANISOTROPIC PARAMETERS (INPUT DATA)             0042
C     TALF44 - MATRIX OF ANISOTROPIC PARAMETERS (INPUT DATA)             0043
C     TEFSTN - MATRIX OF TOTAL EFFECTIVE STRAINS                         0044
C     TEPSN  - TABLE OF STRAIN VALUES DEFINING THE STRESS-STRAIN CURVE   0045
C     TIMK1  - MATRIX OF REFERENCE CREEP TIMES FOR ALL NODES             0046
C     TOTEPS - MATRIX OF TOTAL NODE STRAINS                              0047
C     TSIGN  - TABLE OF STRESS LEVELS DEFINING THE STRESS-STRAIN CURVE   0048
C                                                                        0049
C          * * VARIABLES AND CLUES * *                                   0050
C     ALPHA  - PARAMETER USED IN THE CREEP EQUATIONS                     0051
C     BETA   - PARAMETER USED IN THE CREEP EQUATIONS                     0052
C     E      - MODULUS OF ELASTICITY                                     0053
C     GAMMA  - PARAMETER USED IN THE CREEP EQUATIONS                     0054
C     GNU    - POISSONS RATIO                                            0055
C     K      - THE CYCLE COUNTER                                         0056
C     KERRSW - CLUE USED FOR TEMPORARY INDICATOR BETWEEN LINKS           0057
C     KLU4   - CLUE INDICATING TOTAL COUNT OF MATRICES ON AUXILIARY TAPE 0058
C     KLU5   - CLUE INDICATING MATRICES STILL NOT READ FROM AUXIL. TAPE  0059
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C      KLU6   - CLUE FOR PRINT CONTROL (WHICH CYCLES)                          0060
C      KLU8   - CLUE FOR PRINT CONTROL (WHICH MATRICES)                        0061
C      KLUISO - CLUE FOR ISOTROPIC OR ANISOTROPIC RUN (INPUT)                  0062
C      KSET   - CLUE FOR THE LOAD OR TIME LEVEL CURRENTLY IN USE               0063
C      NA     - 3 TIMES THE NODE COUNT (INPUT)                                 0064
C      NC     - NUMBER OF NODES                                                0065
C      NINCLD - NUMBER OF NON-INCREMENT CYCLES AT EACH LOAD LEVEL (INPUT)      0066
C      PM     - CURRENT LOAD LEVEL                                             0067
C      SHRMOD - SHEAR MODULUS                                                  0068
C      TIME   - CURRENT TIME LEVEL                                             0069
C                                                                              0070
C      OUTPUT - SUBROUTINE TO CONTROL PRINTING AT VARYING LOAD LEVELS          0071
C                                                                              0072
C                                                                              0073
C      ELASTIC UNLOADING (FOLLOWS HOOKES LAW WHEN UNLOADING)                   0074
       COMMON TEFSTN, TOTEPS                                                   0075
       COMMON   KLU4,   KLU5,   KLU6,   KLU8,     NA,     NC,     K          0076
       COMMON KERRSW, NINCLD,   KSET,     PM,      E,    GNU, SHRMOD          0077
       COMMON  ALPHA,   BETA,  GAMMA,   TIME, KLUISO                          0078
       COMMON   PMTM,  KPMTM,    SIM,    SIJ,  TSIGN,  TEPSN                  0079
       COMMON TALF12, TALF23, TALF31, TALF44                                   0080
       COMMON AL1212, AL1223, AL1231, ALFA44                                   0081
       DIMENSION TALF12(11),TALF23(11),TALF31(11),TALF44(11)                   0082
       DIMENSION AL1212(55),AL1223(55),AL1231(55),ALFA44(55)                   0083
       EQUIVALENCE (TIMK1,TEFSTN),(TOTEPS,DELEPK)                              0084
C      THESE FOUR ARRAYS ARE EQUIVALENCED TO SAVE CORE SPACE.                  0085
       DIMENSION TIMK1(55),TEFSTN(55),TOTEPS(165),DELEPK(165)                  0086
       DIMENSION PMTM(10,3),KPMTM(10),SIM(165),SIJ(165,165),TSIGN(11)          0087
       DIMENSION TEPSN(11)                                                     0088
       DIMENSION BL(5),ID(9),NR(3),ND(3),EL(3)                                 0089
       DIMENSION PSTEP(3)                                                      0090
       EQUIVALENCE (ID(1),AD1),(ID(2),AD2),(ID(3),AD3),(ID(4),AD4),(ID(5)      0091
      1,AD5),(ID(6),AD6),(ID(7),AD7),(ID(8),AD8),(ID(9),AD9)                   0092
B      AD1 =  606060623144                                                     0093
B      AD2 =  606060623141                                                     0094
B      AD3 =  606362312745                                                     0095
B      AD4 =  606325476245                                                     0096
B      AD5 =  632143260102                                                     0097
B      AD6 =  632143260203                                                     0098
B      AD7 =  632143260301                                                     0099
B      AD8 =  632143260404                                                     0100
B      AD9 =  606060606060                                                     0101
B      BL(1) = 434621246060                                                    0102
B      BL(2) = 633144256060                                                    0103
B      BL(3) = 632147256060                                                    0104
B      BL(4) = 263145626060                                                    0105
B      BL(5) = 606060606060                                                    0106
       KERRSW = 1                                                              0107
       INTAPE = 7                                                              0108
       KLU5 = 0                                                                0109
       KLU6 = 0                                                                0110
       KLU8 = 10000                                                            0111
       KLUALF   = 0                                                            0112
       PSTEP(1) = 0.0                                                          0113
       PSTEP(2) = 0.0                                                          0114
       PSTEP(3) = 0.0                                                          0115
```

```
      DO 51 J22=1,10                                                    0116
   51 KPMTM(J22) = 0                                                    0117
      LROW = 0                                                          0118
      READ INPUT TAPE 7,1,KLU4,NINCLD,NA,KLU7,KLUISO,ALPHA,BETA,GAMMA,  0119
     1GNU,TA,TB,TC,TD,TE                                                0120
      WRITE OUTPUT TAPE 6,21,TA,TB,TC,TD,TE                             0121
C     KLU7 (INPUT) = 0 TO PRINT PRE-SELECTED MATRICES ON SELECTED CYCLES 0122
C     KLU7 (INPUT) = 1 TO PRINT ALL MATRICES ON SELECTED CYCLES         0123
C     KLU7 (INPUT) = 2 TO PRINT PRE-SELECTED MATRICES ON ALL CYCLES     0124
C     KLU7 (INPUT) = 3-9 TO PRINT ALL MATRICES ON ALL CYCLES            0125
      IF(KLU7)384,384,381                                               0126
  381 IF(KLU7-1)383,383,382                                             0127
  382 KLU6 = 1                                                          0128
C     KLU6 (OUTPUT) = 1 TO PRINT EACH CYCLE, 0 TO PRINT SELECTED CYCLES 0129
      IF(KLU7-2)384,384,383                                             0130
  383 KLU8 = 0                                                          0131
C     KLU8 (OUTPUT) = 0 TO PRINT ALL MATRICES                           0132
C     KLU8 (OUTPUT) = 10000 TO PRINT SELECTED MATRICES                  0133
  384 CONTINUE                                                          0134
      IF(KLU4)302,302,301                                               0135
  301 PRINT 17                                                          0136
      PAUSE 11111                                                       0137
C     TEST INPUT TAPE TO DISTINGUISH GISMO BCD FROM INELASTIC PROG. SAVE 0138
      READ INPUT TAPE 9,31,LTAPE                                        0139
      REWIND 9                                                          0140
C     SUBROUTINE FILTAP POSITIONS A TAPE AT THE FIRST RECORD OF ANY FILE 0141
      CALL FILTAP(9,2)                                                  0142
      IF(LTAPE-21)343,343,344                                           0143
  344 CONTINUE                                                          0144
C     SAVED TAPE FROM PREVIOUS RUN OF INELASTIC PLATE                   0145
C     THIS RUN IS A CONTINUATION                                        0146
      CALL CHAIN (2,3)                                                  0147
  343 CONTINUE                                                          0148
C     GISMO FORMAT BCD TAPE                                             0149
C     THIS RUN IS A NEW ONE                                             0150
      KLU5 = KLU4                                                       0151
      INTAPE = 9                                                        0152
  302 CONTINUE                                                          0153
C     KLUISO   = 0  FOR ISOTROPIC RUN                                   0154
C     KLUISO   = 1  FOR ANISOTROPIC RUN                                 0155
      WRITE OUTPUT TAPE 6,9                                             0156
      IF(KLUISO)380,380,379                                             0157
  379 CONTINUE                                                          0158
      WRITE OUTPUT TAPE 6,10                                            0159
  380 CONTINUE                                                          0160
   52 READ INPUT TAPE 7,2,TEMP1,TEMP2,TEMP3,TEMP4                       0161
      LROW = LROW + 1                                                   0162
      DO 53 J22=1,5                                                     0163
8     IF(TEMP1*(-BL(J22)))53,54,53                                      0164
   53 CONTINUE                                                          0165
C     BAD CONTROL CARD                                                  0166
      WRITE OUTPUT TAPE 6,12,LROW                                       0167
      WRITE OUTPUT TAPE 6,2,TEMP1,TEMP2,TEMP3,TEMP4                     0168
      CALL EXIT                                                         0169
   54 GO TO(55,55,55,60,60),J22                                         0170
   55 IF(LROW-10)56,56,53                                               0171
```

```
C      KPMTM( N ) = 1 FOR A LOAD STEP                                    0172
C      KPMTM( N ) = 2 FOR A TIME STEP                                    0173
C      KPMTM( N ) = 3 TO DUMP MEMORY INTO A SAVE TAPE                    0174
C      PMTM (N,1) = UPPER LIMIT OF STEP                                  0175
C      PMTM (N,2) = INTERVAL (INCREMENT) FOR CALCULATION                 0176
C      PMTM (N,3) = INTERVAL (INCREMENT) FOR PRINT OUTPUT                0177
C                                                                        0178
   56 KPMTM(LROW) = J22                                                  0179
      PMTM(LROW,1)= TEMP2                                               0180
      TEMP5       = TEMP2-PSTEP(J22)                                    0181
      PMTM(LROW,2)= SIGNF(TEMP3,TEMP5)                                  0182
      PSTEP(J22)  = TEMP2                                               0183
      IF(PMTM(LROW,2))57,63,57                                          0184
   63 GO TO(995,995,57),J22                                             0185
   57 CONTINUE                                                          0186
      PMTM(LROW,3)= TEMP4                                               0187
      GO TO 52                                                          0188
   60 CONTINUE                                                          0189
      DO 361 J23 = 1,10                                                 0190
      IF(KPMTM(J23))361,361,362                                         0191
  362 IF(KPMTM(J23)-4)363,361,361                                       0192
  363 J24 = KPMTM(J23)                                                  0193
      GO TO (364,366,368),J24                                           0194
  364 WRITE OUTPUT TAPE 6,22,PMTM(J23,2),PMTM(J23,1)                    0195
      IF(PMTM(J23,3))361,361,365                                        0196
  365 WRITE OUTPUT TAPE 6,23,PMTM(J23,3)                                0197
      GO TO 361                                                         0198
  366 WRITE OUTPUT TAPE 6,24,PMTM(J23,2),PMTM(J23,1)                    0199
      IF(PMTM(J23,3))361,361,367                                        0200
  367 WRITE OUTPUT TAPE 6,26,PMTM(J23,3)                                0201
      GO TO 361                                                         0202
  368 WRITE OUTPUT TAPE 6,27                                            0203
  361 CONTINUE                                                          0204
      WRITE OUTPUT TAPE 6,28,NINCLD                                     0205
      WRITE OUTPUT TAPE 6,25,ALPHA,BETA,GAMMA                           0206
      IF(GNU)86,87,87                                                   0207
   86 GNU = .3                                                          0208
   87 CONTINUE                                                          0209
      IF(NA)999,91,92                                                   0210
   91 NA  = 165                                                         0211
   92 IF(NA-165)93,93,999                                               0212
   93 CONTINUE                                                          0213
   96 IF(NA-3*(NA/3))999,97,999                                         0214
   97 NC  = NA/3                                                        0215
      IF(KLU4)304,304,303                                               0216
  303 IF(KLU4-4)306,306,994                                             0217
  306 CONTINUE                                                          0218
      INTAPE = 9                                                        0219
      KLU5 = KLU5 - 1                                                   0220
  108 READ INPUT TAPE INTAPE, 3,NAME,NROWS,NCOLS,NFORM                  0221
      DO 110 I21=1,7                                                    0222
      IF(NAME-ID(I21))110,311,110                                       0223
  110 CONTINUE                                                          0224
C     BAD INPUT - MATRIX NAME NOT ACCEPTABLE                            0225
      GO TO 997                                                         0226
  311 GO TO (321,322,323,324,325,326,327,328,111),I21                   0227
  321 DO 331 I1 =1,165
```

```
  331 SIM(I1) = 0.0                                                      0228
      WRITE OUTPUT TAPE 6,20,NAME,NROWS,NCOLS,INTAPE                     0229
      GO TO 111                                                          0230
  322 DO 332 I1 =1,165                                                   0231
      DO 332 I2 =1,165                                                   0232
  332 SIJ(I1,I2) = 0.0                                                   0233
      GO TO 111                                                          0234
  323 DO 333 I1 = 1,11                                                   0235
  333 TSIGN(I1) = 0.0                                                    0236
      GO TO 111                                                          0237
  324 DO 334 I1 = 1,11                                                   0238
  334 TEPSN(I1) = 0.0                                                    0239
      GO TO 111                                                          0240
  325 DO 335 I1 = 1,11                                                   0241
  335 TALF12(I1)= 0.0                                                    0242
      KLUALF    = KLUALF + 1                                             0243
      GO TO 111                                                          0244
  326 DO 336 I1 = 1,11                                                   0245
  336 TALF23(I1)= 0.0                                                    0246
      KLUALF    = KLUALF + 2                                             0247
      GO TO 111                                                          0248
  327 DO 337 I1 = 1,11                                                   0249
  337 TALF31(I1)= 0.0                                                    0250
      KLUALF    = KLUALF + 4                                             0251
      GO TO 111                                                          0252
  328 DO 338 I1 = 1,11                                                   0253
  338 TALF44(I1)= 0.0                                                    0254
      KLUALF    = KLUALF +8                                              0255
      GO TO 111                                                          0256
  111 GO TO (112,112,112,112,112,112,112,112,150),121                    0257
  112 READ INPUT TAPE INTAPE,4,(NR(I22),ND(I22),EL(I22),I22=1,3)         0258
      IF(NR(1))996,109,113                                               0259
  109 IF(KLU5)308,308,306                                                0260
  308 INTAPE = 7                                                         0261
      GO TO 108                                                          0262
  304 GO TO 308                                                          0263
  113 GO TO (121,122,125,126,201,202,203,204,150),121                    0264
C     READ IN ARRAY SIGMA-IM                                             0265
  121 MROW = NR(1)                                                       0266
      WRITE OUTPUT TAPE 6,4,(NR(I22),ND(I22),EL(I22),I22=1,3)            0267
      SIM   (MROW) = EL(1)                                               0268
      IF(EL(2))127,128,127                                               0269
  127 MROW = NR(2)                                                       0270
      SIM   (MROW) = EL(2)                                               0271
  128 IF(EL(3))129,112,129                                               0272
  129 MROW = NR(3)                                                       0273
      SIM   (MROW) = EL(3)                                               0274
      GO TO 112                                                          0275
C     READ IN ARRAY SIGMA-IJ                                             0276
  122 MROW = NR(1)                                                       0277
      MCOL = ND(1)                                                       0278
      SIJ   (MROW,MCOL) = EL(1)                                          0279
      IF(EL(2))130,131,130                                               0280
  130 MROW = NR(2)                                                       0281
      MCOL = ND(2)                                                       0282
      SIJ   (MROW,MCOL) = EL(2)                                          0283
```

```
  131 IF(EL(3))132,112,132                                              0284
  132 MROW = NR(3)                                                      0285
      MCOL = ND(3)                                                      0286
      SIJ   (MROW,MCOL) = EL(3)                                         0287
      GO TO 112                                                         0288
C     READ IN ARRAY TSIGN (TABLE OF SIGMA BAR N)                        0289
  125 MROW = NR(1)                                                      0290
      TSIGN(MROW) = EL(1)                                               0291
      IF(EL(2))139,140,139                                              0292
  139 MROW = NR(2)                                                      0293
      TSIGN(MROW) = EL(2)                                               0294
  140 IF(EL(3))141,112,141                                              0295
  141 MROW = NR(3)                                                      0296
      TSIGN(MROW) = EL(3)                                               0297
      GO TO 112                                                         0298
C     READ IN ARRAY TEPSN (TABLE OF EPSILON BAR N)                      0299
  126 MROW = NR(1)                                                      0300
      TEPSN(MROW) = EL(1)                                               0301
      IF(EL(2))142,143,142                                              0302
  142 MROW = NR(2)                                                      0303
      TEPSN(MROW) = EL(2)                                               0304
  143 IF(EL(3))144,112,144                                              0305
  144 MROW = NR(3)                                                      0306
      TEPSN(MROW) = EL(3)                                               0307
      GO TO 112                                                         0308
C     READ IN ALPHA TABLES                                              0309
  201 MROW = NR(1)                                                      0310
      TALF12(MROW) = EL(1)                                              0311
      IF(EL(2))221,222,221                                              0312
  221 MROW = NR(2)                                                      0313
      TALF12(MROW) = EL(2)                                              0314
  222 IF(EL(3))223,112,223                                              0315
  223 MROW = NR(3)                                                      0316
      TALF12(MROW) = EL(3)                                              0317
      GO TO 112                                                         0318
  202 MROW = NR(1)                                                      0319
      TALF23(MROW) = EL(1)                                              0320
      IF(EL(2))224,225,224                                              0321
  224 MROW = NR(2)                                                      0322
      TALF23(MROW) = EL(2)                                              0323
  225 IF(EL(3))226,112,226                                              0324
  226 MROW = NR(3)                                                      0325
      TALF23(MROW) = EL(3)                                              0326
      GO TO 112                                                         0327
  203 MROW = NR(1)                                                      0328
      TALF31(MROW) = EL(1)                                              0329
      IF(EL(2))227,228,227                                              0330
  227 MROW = NR(2)                                                      0331
      TALF31(MROW) = EL(2)                                              0332
  228 IF(EL(3))229,112,229                                              0333
  229 MROW = NR(3)                                                      0334
      TALF31(MROW) = EL(3)                                              0335
      GO TO 112                                                         0336
  204 MROW = NR(1)                                                      0337
      TALF44(MROW) = EL(1)                                              0338
      IF(EL(2))230,231,230                                              0339
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  230 MROW = NR(2)                                                        0340
      TALF44(MROW) = EL(2)                                                0341
  231 IF(EL(3))232,112,232                                                0342
  232 MROW = NR(3)                                                        0343
      TALF44(MROW) = EL(3)                                                0344
      GO TO 112                                                           0345
  150 CONTINUE                                                            0346
      IF(KLUISO)165,165,161                                               0347
  161 IF(KLUALF-15)162,167,162                                            0348
  162 WRITE OUTPUT TAPE 6,32,KLUALF                                       0349
      IF(KLU4)164,164,163                                                 0350
C     SUBROUTINE RUN REWINDS AND UNLOADS THE DESIGNATED TAPE              0351
  163 CALL RUN(9)                                                         0352
  164 CALL EXIT                                                           0353
  165 DO 166 I = 1,11                                                     0354
      TALF12(I)= 0.5                                                      0355
      TALF23(I)= 0.5                                                      0356
      TALF31(I)= 0.5                                                      0357
  166 TALF44(I)= 1.0                                                      0358
  167 CONTINUE                                                            0359
      E = TSIGN(2)/TEPSN(2)                                               0360
   81 SHRMOD = E/(2.*(1.+GNU))                                            0361
      WRITE OUTPUT TAPE 6,5,E,SHRMOD,GNU                                  0362
      WRITE OUTPUT TAPE 6,6                                               0363
      DO 149 I1=1,11                                                      0364
  149 WRITE OUTPUT TAPE 6,7,I1,TSIGN(I1),TEPSN(I1),TALF12(I1),TALF23(I1)  0365
     1,TALF31(I1),TALF44(I1)                                              0366
      K   = 0                                                             0367
      PM  = 0.0                                                           0368
      IF(KLU4)152,152,209                                                 0369
C     SUBROUTINE RUN REWINDS AND UNLOADS THE DESIGNATED TAPE              0370
  209 CALL RUN(9)                                                         0371
  151 PRINT 19                                                            0372
  152 CALL CHAIN(4,3)                                                     0373
  994 WRITE OUTPUT TAPE 6,18,KLU4                                         0374
      CALL EXIT                                                           0375
  995 WRITE OUTPUT TAPE 6,16,LKOW                                         0376
      CALL EXIT                                                           0377
  996 CONTINUE                                                            0378
      WRITE OUTPUT TAPE 6,14,NAME                                         0379
      WRITE OUTPUT TAPE 6,4,(NR(I22),ND(I22),EL(I22),I22=1,3)             0380
      CALL EXIT                                                           0381
  997 CONTINUE                                                            0382
      WRI E OUTPUT TAPE 6, 13, NAME                                       0383
      CALL EXIT                                                           0384
  999 WRITE OUTPUT TAPE 6,11,NA                                           0385
      CALL EXIT                                                           0386
    1 FORMAT(I1,2I3,2I1,1X,2E10.3,2F5.2,10X,5A6)                          0387
    2 FORMAT(6X,A4,E10.6,E10.1,E10.1)                                     0388
    3 FORMAT(11X,A6,1X,2I3,8X,I1)                                         0389
    4 FORMAT(3(1X,2I3,E16.8)  )                                           0390
    5 FORMAT(26H1 MODULUS OF ELASTICITY = ,F11.0,4H PSI,6X,16HSHEAR MODU  0391
     1LUS = ,F11.0,4H PSI,6X,5HNU = ,F6.3)                                0392
    6 FORMAT(41HO TABLE OF VALUES FOR STRESS-STRAIN CURVE  //             0393
     15X,29HPOINT  STRESS LEVEL     STRAIN,7X,8HALPHA 12,7X,8HALPHA 23,/X 0394
     1,8HALPHA 31,7X,8HALPHA 44/16X,3HPSI,9X,7HIN./IN.//)                 0395
```

```
    7 FORMAT(6X,I3,2X,F11.2,5(3X,F12.6))                                  0396
    9 FORMAT(//79H THIS   ISOTROPIC RUN USES ELASTIC UNLOADING (HOOKES L  0397
     1AW) WITH STRAIN HARDENING)                                          0398
   10 FORMAT(1H+,5X,2HAN)                                                 0399
   11 FORMAT(10H ERROR NA=I4)                                             0400
   12 FORMAT(26H ERROR- INCREMENT CARD NO.,I3,5H N.G.)                    0401
   13 FORMAT(14H ERROR-MATRIX ,A6)                                        0402
   14 FORMAT(17H ERROR-NEG.INDEX A6)                                      0403
   16 FORMAT(33H ERROR-NO INTERVAL-INCREMENT CARD,I3)                     0404
   17 FORMAT(51HO PAUSE 11111 TO MOUNT INPUT DATA TAPE ON DRIVE A-5////)  0405
   18 FORMAT(7H ERROR-I5,18H INPUT MATRICES NG)                           0406
   19 FORMAT(89H DEMOUNT AND SAVE TAPE A-5. DATA HAS BEEN READ INTO MEMO  0407
     1RY. THE PROGRAM CONTINUES TO RUN.//)                                0408
   20 FORMAT(//5X,7HMATRIX ,A6,1X,I3,6H ROWS X ,I3,19H COLUMNS FROM TAPE  0409
     1 ,I2)                                                               0410
   21 FORMAT(1H1,29X,5A6)                                                 0411
   22 FORMAT(5X,16HLOAD INCREMENTS ,F9.2,11H POUNDS TO ,F10.2,7H POUNDS)  0412
   23 FORMAT(1H+,61X,19HPRINT OUTPUT EVERY ,F9.2,7H POUNDS)               0413
   24 FORMAT(5X,16HTIME INCREMENTS ,F9.4,11H HOURS TO  ,F10.4,6H HOURS)   0414
   25 FORMAT(/ 9H ALPHA = ,E10.3,5X,7HBETA = ,E10.3,5X,8HGAMMA = ,E10.3)  0415
   26 FORMAT(1H+,61X,19HPRINT OUTPUT EVERY ,F9.4,6H HOURS)                0416
   27 FORMAT(5X,35HSTORE MEMORY ON TAPE A-6, THEN EXIT)                   0417
   28 FORMAT(//5X,I3,48H NON-INCREMENT CYCLES AT EACH LOAD OR TIME LEVEL  0418
     1)                                                                   0419
   31 FORMAT(10X,I2)                                                      0420
   32 FORMAT(5X,I2,65H ALPHA TABLES WERE READ IN OF 4 REQUIRED. CHECK YO  0421
     1UR INPUT CARDS.)                                                    0422
      END(1,1,0,0,0,0,1,1,0,1,0,0,0,0,0)

*     CHAIN(2,3)                                                         0424
*     LIST8                                                              0425
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451282  GRUMMAN AIRCRAFT ENGINEERING CORP.   DECK NO. 45128   LINK 2

C451282  MATRIX ANALYSIS OF INELASTIC PLATE - LINK 2 - WITH CREEP          0427
C     THIS LINK READS IN A SAVED BINARY TAPE - FIRST PART                  0428
      COMMON TEFSTN, TOTEPS                                                0429
      COMMON   KLU4,   KLU5,   KLU6,   KLU8,     NA,     NC,     K        0430
      COMMON KERRSW, NINCLD,   KSET,     PM,      E,    GNU, SHRMOD        0431
      COMMON  ALPHA,   BETA,  GAMMA,   TIME, KLUISO                        0432
      COMMON   PMTM,  KPMTM,    SIM,    SIJ,  TSIGN,  TEPSN                0433
      COMMON TALF12, TALF23, TALF31, TALF44                                0434
      COMMON AL1212, AL1223, AL1231, ALFA44                                0435
      DIMENSION TALF12(11),TALF23(11),TALF31(11),TALF44(11)                0436
      DIMENSION AL1212(55),AL1223(55),AL1231(55),ALFA44(55)                0437
      COMMON  SIGUK,  EPSUK                                                0438
      EQUIVALENCE (TIMK1,TEFSTN),(TOTEPS,DELEPK)                           0439
C     THESE FOUR ARRAYS ARE EQUIVALENCED TO SAVE CORE SPACE.               0440
      DIMENSION TIMK1(55),TEFSTN(55),TOTEPS(165),DELEPK(165)               0441
      DIMENSION PMTM(10,3),KPMTM(10),SIM(165),SIJ(165,165),TSIGN(11)       0442
      DIMENSION TEPSN(11),SIGUK(165),EPSUK(165)                            0443
      DIMENSION BL(5)                                                      0444
      DIMENSION PSTEP(3)                                                   0445
      READ  TAPE  9,KLU4,KLU5,  LA,  LB,NA,NC,K,KERRSW,    LC,KSET,PM,E,  0446
     1GNU,SHRMOD,ALPHA,BETA,GAMMA,TIME,KLUISO                              0447
C     KLU6 = 1 WILL PRINT EVERY CYCLE, 0 WILL PRINT ONLY SELECTED CYCLES   0448
C     KLU8 = 0 WILL PRINT ALL MATRICES, NON-ZERO PRINTS SELECTED MATRICES  0449
C     NINCLD (NON-INCREMENT CYCLES AT EACH LOAD LEVEL) ACCEPTED FROM       0450
C     INPUT CONTROL CARD                                                   0451
      WRITE OUTPUT TAPE 6,9                                                0452
      IF(KLUISO)380,380,379                                                0453
  379 CONTINUE                                                             0454
      WRITE OUTPUT TAPE 6,10                                               0455
  380 CONTINUE                                                             0456
      NCLU = 1                                                             0457
      KERRSW = 2                                                           0458
B     BL(1) = 434621246060                                                 0459
B     BL(2) = 633144256060                                                 0460
B     BL(3) = 632147256060                                                 0461
B     BL(4) = 263145626060                                                 0462
B     BL(5) = 606060606060                                                 0463
      DO 51 I42 = 1,10                                                     0464
   51 KPMTM(I42) = 0                                                       0465
      PSTEP(1) = 0.0                                                       0466
      PSTEP(2) = 0.0                                                       0467
      PSTEP(3) = 0.0                                                       0468
      LROW = 0                                                             0469
   52 READ INPUT TAPE 7,2,TEMP1,TEMP2,TEMP3,TEMP4                          0470
      LROW = LROW + 1                                                      0471
      DO 53 J22=1,5                                                        0472
B     IF(TEMP1*(-BL(J22)))53,54,53                                         0473
   53 CONTINUE                                                             0474
C     BAD CONTROL CARD                                                     0475
      WRITE OUTPUT TAPE 6,12,LROW                                          0476
      WRITE OUTPUT TAPE 6,2,TEMP1,TEMP2,TEMP3,TEMP4                        0477
      CALL EXIT                                                            0478
   54 GO TO(55,55,55,60,60),J22                                            0479
   55 IF(LROW-10)56,56,53                                                  0480
C     KPMTM( N ) = 1 FOR A LOAD STEP                                       0481
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C     KPMTM( N ) = 2 FOR A TIME STEP                                    0482
C     KPMTM( N ) = 3 TO DUMP MEMORY INTO A SAVE TAPE                    0483
C     PMTM (N,1) = UPPER LIMIT OF STEP                                  0484
C     PMTM (N,2) = INTERVAL (INCREMENT) FOR CALCULATION                 0485
C     PMTM (N,3) = INTERVAL (INCREMENT) FOR PRINT OUTPUT                0486
   56 KPMTM(LROW) = J22                                                 0487
      NCLU = 2                                                          0488
      KSET = 1                                                          0489
      PMTM(LROW,1)= TEMP2                                               0490
      TEMP5       = TEMP2-PSTEP(J22)                                    0491
      PMTM(LROW,2)= SIGNF(TEMP3,TEMP5)                                  0492
      PSTEP(J22)  = TEMP2                                               0493
      IF(PMTM(LROW,2))57,63,57                                          0494
   63 GO TO (995,995,57),J22                                            0495
   57 CONTINUE                                                          0496
      PMTM(LROW,3)= TEMP4                                               0497
      GO TO 52                                                          0498
   60 CONTINUE                                                          0499
      GO TO (61,62),NCLU                                                0500
   62 CONTINUE                                                          0501
      READ  TAPE  9,JUNK                                                0502
      READ  TAPE  9,JUNK                                                0503
      GO TO 64                                                          0504
   61 CONTINUE                                                          0505
      READ  TAPE  9,PMTM                                                0506
      READ  TAPE  9,KPMTM                                               0507
   64 CONTINUE                                                          0508
      WRITE OUTPUT TAPE 6,1                                             0509
      DO 361 J23 = 1,10                                                 0510
      IF(KPMTM(J23))361,361,362                                         0511
  302 IF(KPMTM(J23)-4)363,361,301                                       0512
  363 J24 = KPMTM(J23)                                                  0513
      GO TO (364,366,368),J24                                           0514
  364 WRITE OUTPUT TAPE 6,22,PMTM(J23,2),PMTM(J23,1)                    0515
      IF(PMTM(J23,3))361,361,305                                        0516
  365 WRITE OUTPUT TAPE 6,23,PMTM(J23,3)                                0517
      GO TO 361                                                         0518
  366 WRITE OUTPUT TAPE 6,24,PMTM(J23,2),PMTM(J23,1)                    0519
      IF(PMTM(J23,3))361,361,307                                        0520
  367 WRITE OUTPUT TAPE 6,25,PMTM(J23,3)                                0521
      GO TO 361                                                         0522
  368 WRITE OUTPUT TAPE 6,27                                            0523
  361 CONTINUE                                                          0524
      WRITE OUTPUT TAPE 6,26,NINCLD                                     0525
      READ TAPE   9,SIM                                                 0526
      READ TAPE   9,SIJ                                                 0527
      READ TAPE   9,TSIGN                                               0528
      READ TAPE   9,TEPSN                                               0529
      READ  TAPE  9,TALF12                                              0530
      READ  TAPE  9,TALF23                                              0531
      READ  TAPE  9,TALF31                                              0532
      READ  TAPE  9,TALF44                                              0533
      READ  TAPE  9,AL1212                                              0534
      READ  TAPE  9,AL1223                                              0535
      READ  TAPE  9,AL1231                                              0536
      READ  TAPE  9,ALFA44                                              0537
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      READ TAPE   9,SIGUK                                                    0538
      READ TAPE   9,EPSUK                                                    0539
      CALL CHAIN (3,3)                                                       0540
  995 WRITE OUTPUT TAPE 6,16,LKUN                                            0541
      CALL EXIT                                                              0542
    1 FORMAT(1HO,29X,37HCONTINUATION RUN - INELASTIC ANALYSIS)               0543
    2 FORMAT(6X,A4,E10.6,E10.1,E10.1)                                        0544
    9 FORMAT(//79H THIS   ISOTROPIC RUN USES ELASTIC UNLOADING (HOOKES L     0545
     1AW) WITH STRAIN HARDENING)                                             0546
   10 FORMAT(1H+,5X,2HAN)                                                    0547
   12 FORMAT(26H ERROR- INCREMENT CARD NO.,I3,5H N.G.)                       0548
   16 FORMAT(33H ERROR-NO INTERVAL-INCREMENT CARD,I3)                        0549
   22 FORMAT(5X,16HLOAD INCREMENTS ,F9.2,11H POUNDS TO ,F10.2,7H POUNDS)     0550
   23 FORMAT(1H+,61X,19HPRINT OUTPUT EVERY ,F9.2,7H POUNDS)                  0551
   24 FORMAT(5X,16HTIME INCREMENTS ,F9.4,11H HOURS TO  ,F10.4,6H HOURS)      0552
   25 FORMAT(1H+,61X,19HPRINT OUTPUT EVERY ,F9.4,6H HOURS)                   0553
   26 FORMAT(//5X,I3,48H NON-INCREMENT CYCLES AT EACH LOAD OR TIME LEVEL     0554
     1)                                                                      0555
   27 FORMAT(5X,35HSTORE MEMORY ON TAPE A-6, THEN EXIT)                      0556
      END(1,1,0,0,0,0,1,1,0,1,0,0,0,0,0)

*     CHAIN(3,3)                                                           0558
*     LIST8                                                                0559
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 451283  GRUMMAN AIRCRAFT ENGINEERING CORP.   DECK NO. 45128   LINK 3

C451283  MATRIX ANALYSIS OF INELASTIC PLATE   LINK 3 - WITH CREEP          0561
C      THIS IS THE SECOND HALF OF OLD LINK 2                               0562
C      THIS LINK READS IN A SAVED BINARY TAPE - SECOND PART                0563
       COMMON TEFSTN, TOTEPS                                               0564
       COMMON   KLU4,   KLU5,   KLU6,   KLU8,     NA,     NC,     K       0565
       COMMON KERRSW, NINCLD,   KSET,     PM,      E,    GNU, SHRMOD       0566
       COMMON  ALPHA,   BETA,  GAMMA,   TIME, KLUISO                       0567
       COMMON   PMTM,  KPMTM,    SIM,    SIJ,  TSIGN,  TEPSN               0568
       COMMON TALF12, TALF23, TALF31, TALF44                               0569
       COMMON AL1212, AL1223, AL1231, ALFA44                               0570
       DIMENSION TALF12(11),TALF23(11),TALF31(11),TALF44(11)               0571
       DIMENSION AL1212(55),AL1223(55),AL1231(55),ALFA44(55)               0572
       COMMON  SIGUK,  EPSUK                                               0573
       COMMON SGBARN, SGBARP, SGBARM, EPBARN, EPBARP, DELEPN               0574
       COMMON  EPCNK,  EPCNP                                               0575
       EQUIVALENCE (TIMK1,TEFSTN),(TOTEPS,DELEPK)                          0576
C      THESE FOUR ARRAYS ARE EQUIVALENCED TO SAVE CORE SPACE.              0577
       DIMENSION TIMK1(55),TEFSTN(55),TOTEPS(165),DELEPK(165)              0578
       DIMENSION PMTM(10,3),KPMTM(10),SIM(165),SIJ(165,165),TSIGN(11)      0579
       DIMENSION TEPSN(11),SIGUK(165),EPSUK(165),SGBARN(55),SGBARP(55)     0580
       DIMENSION SGBARM(55),EPBARN(55),EPBARP(55),DELEPN(55),EPCNK(55)     0581
       DIMENSION EPCNP(55)                                                 0582
       READ TAPE   9,SGBARN                                                0583
       READ  TAPE  9,SGBARP                                                0584
       READ  TAPE  9,SGBARM                                                0585
       READ TAPE   9,EPBARN                                                0586
       READ TAPE   9,EPBARP                                                0587
       READ TAPE   9,DELEPN                                                0588
       READ  TAPE  9,EPCNK                                                 0589
       READ  TAPE  9,EPCNP                                                 0590
C      SUBROUTINE RUN REWINDS AND UNLOADS THE DESIGNATED TAPE              0591
       CALL RUN(9)                                                         0592
       PRINT 31                                                            0593
       WRITE OUTPUT TAPE 6,5,E,SHRMOD,GNU                                  0594
       WRITE OUTPUT TAPE 6,6                                               0595
       DO 149 I1=1,11                                                      0596
  149  WRITE OUTPUT TAPE 6,7,I1,TSIGN(I1),TEPSN(I1),TALF12(I1),TALF23(I1)  0597
      1,TALF31(I1),TALF44(I1)                                              0598
       CALL CHAIN (4,3)                                                    0599
     5 FORMAT(26HO MODULUS OF ELASTICITY = ,F11.0,4H PSI,6X,16HSHEAR MODU  0600
      1LUS = ,F11.0,4H PSI,6X,5HNU = ,F6.3)                                0601
     6 FORMAT(41HO TABLE OF VALUES FOR STRESS-STRAIN CURVE  //             0602
      15X,29HPOINT  STRESS LEVEL    STRAIN,7X,8HALPHA 12,7X,8HALPHA 23,7X  0603
      1,8HALPHA 31,7X,8HALPHA 44/10X,3HPSI,9X,7HIN./IN.//)                 0604
     7 FORMAT(6X,I3,2X,F11.2,5(3X,F12.8))                                  0605
    31 FORMAT(118HOIF NECESSARY TO STOP THIS RUN BECAUSE IT EXCEEDS THE T  0606
      1IME ESTIMATE, PUT A RING IN THE SAVE TAPE ON A-5 AND CHANGE IT /     0607
      256H TO DRIVE A-6 TO UPDATE IT, THEN PUT SENSE SWITCH 6 ON. //)       0608
       END(1,1,0,0,0,0,1,1,0,1,0,0,0,0,0)


*                                                                          0610
*      SAVE TAPE 8-1 UNLESS ON-LINE PRINT SAYS IT HAS BEEN PROCESSED       0611
*                                                                          0612
*      PUT SENSE SWITCH 6 ON TO END THE RUN IF IT EXCEEDS THE TIME ESTIM.  0613
*                                                                          0614
*      IF PAUSE 1 OCCURS (SENSE SWITCH 6 OR INTERNAL CONTROL), MOUNT A     0615
*      BLANK TAPE ON A-6. THIS TAPE WILL HAVE RESTART DATA WRITTEN ON IT,  0616
*      AND MUST BE SAVED.                                                  0617
*      CHAIN(4,3)                                                          0618
*      LIST8                                                               0619
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C451284  MATRIX ANALYSIS OF INELASTIC PLATE - LINK 4 - WITH CREEP          0621
C      THIS LINK DOES THE CALCULATION AND WRITES PRINT OUTPUT              0622
       COMMON TEFSTN, TOTEPS                                              0623
       COMMON   KLU4,   KLU5,   KLU6,   KLU8,     NA,     NC,     K      0624
       COMMON KERRSW, NINCLD,   KSET,     PM,      E,    GNU, SHRMOD      0625
       COMMON  ALPHA,   BETA,  GAMMA,   TIME, KLUISO                      0626
       COMMON   PMTM,  KPMTM,    SIM,    SIJ,  TSIGN,  TEPSN              0627
       COMMON TALF12, TALF23, TALF31, TALF44                              0628
       COMMON AL1212, AL1223, AL1231, ALFA44                              0629
       DIMENSION TALF12(11),TALF23(11),TALF31(11),TALF44(11)              0630
       DIMENSION AL1212(55),AL1223(55),AL1231(55),ALFA44(55)              0631
       COMMON  SIGUK,  EPSUK                                              0632
       COMMON SGBARN, SGBARP, SGBARM, EPBARN, EPBARP, DELEPN              0633
       COMMON  EPCNK,  EPCNP                                              0634
       EQUIVALENCE (TIMK1,TEFSTN),(TOTEPS,DELEPK)                         0635
C      THESE FOUR ARRAYS ARE EQUIVALENCED TO SAVE CORE SPACE.             0636
C      THEY ARE NOT THE SAME, BUT ARE NOT NEEDED AT THE SAME TIME         0637
C      AND DO NOT CARRY FROM  CYCLE TO CYCLE.                             0638
       DIMENSION TIMK1(55),TEFSTN(55),TOTEPS(165),DELEPK(165)             0639
       DIMENSION PMTM(10,3),KPMTM(10),SIM(165),SIJ(165,165),TSIGN(11)     0640
       DIMENSION TEPSN(11),SIGUK(165),EPSUK(165),SGBARN(55),SGBARP(55)    0641
       DIMENSION SGBARM(55),EPBARN(55),EPBARP(55),DELEPN(55),EPCNK(55)    0642
       DIMENSION EPCNP(55)                                                0643
       KSET     = KSET                                                    0644
       KERRSW   = KERRSW                                                  0645
       GO TO (418,419),KERRSW                                             0646
C      INITIALIZE WORK AREAS                                              0647
  418  DO 102 I1 = 1,165                                                  0648
       EPSUK (I1) = 0.0                                                   0649
  102  SIGUK(I1) = 0.0                                                    0650
       DO 103 I1 = 1,55                                                   0651
       DELEPN(I1) = 0.0                                                   0652
       SGBARN(I1) = 0.0                                                   0653
       SGBARP(I1) = 0.0                                                   0654
       SGBARM(I1) = 0.0                                                   0655
       EPBARP(I1) = 0.0                                                   0656
       EPCNK(I1) = 0.0                                                    0657
       EPCNP(I1) = 0.0                                                    0658
       AL1212(I1)= 2.*TALF12(2)                                           0659
       AL1223(I1)= TALF12(2) + TALF23(2)                                  0660
       AL1231(I1)= TALF12(2) + TALF31(2)                                  0661
       ALFA44(I1)= TALF44(2)                                              0662
  103  EPBARN(I1) =0.0                                                    0663
       KSET = 1                                                           0664
       TIME = 0.0                                                         0665
  419  CONTINUE                                                           0666
       REWIND 8                                                           0667
       KSET     = KSET                                                    0668
       KERRSW   = KERRSW                                                  0669
       NINCPM = NINCLD + 2                                                0670
       NINTOT = NINCLD + 1                                                0671
  151  K    = K + 1                                                       0672
       KLU2 = 1                                                           0673
C      IF KLU2 = 0, THE CYCLE OF OPERATIONS WILL BE PRINTED               0674
C      KLU6 = 1 WILL PRINT EVERY CYCLE, 0 WILL PRINT ONLY SELECTED CYCLES 0675
```

```
C      KLU8 = 0 WILL PRINT ALL MATRICES, NON-ZERO PRINTS SELECTED ONES
       IF(KLU6)248,248,249
  249  KLU2 = 0
  248  CONTINUE
       IF(K-1)270,270,271
  270  KLU2 = 0
  271  CONTINUE
       GO TO (416,417),KERRSW
  417  KLU2 = 0
       KERRSW = 1
  416  CONTINUE
C      KSET IS THE ROW OF KPMTM OR PMTM BEING USED (CURRENT LOAD OR
C      TIME LEVEL)
  300  IF(KSET-11)301,319,319
  301  IF(KPMTM(KSET))302,302,303
  302  KSET = KSET + 1
       GO TO 300
  303  IF(KPMTM(KSET)-4)304,302,302
C      VARIABLE (J24) INDICATES A LOAD CYCLE (1), A TIME CYCLE (2),
C      OR WRITE MEMORY ON A SAVE TAPE (3)
  304  J24  = KPMTM(KSET)
       KSET     = KSET
       KERRSW   = KERRSW
       IF(PMTM(KSET,2))342,342,341
  341  GO TO (305,315,423),J24
  342  GO TO (345,355,423),J24
  305  IF(PMTM(KSET,1)-(PM+PMTM(KSET,2)))306,307,308
  345  IF(PMTM(KSET,1)-(PM+PMTM(KSET,2)))308,307,306
  307  IF(NINCLD)309,309,308
  309  KLU2  = 0
  308  DELPM = PMTM(KSET,2)
       GO TO 154
  306  IF(NINCPM-2)316,316,154
  316  KLU2  = 0
       GO TO 302
  315  IF(PMTM(KSET,1)-(TIME+PMTM(KSET,2)))306,317,318
  355  IF(PMTM(KSET,1)-(TIME+PMTM(KSET,2)))318,317,306
  317  IF(NINCLD)314,314,318
  314  KLU2 = 0
  318  DELTIM= PMTM(KSET,2)
       GO TO 330
  319  IF(KLU2)205,205,320
  320  KLU2  = 0
       KSET  = KSET - 1
       IF(KSET)205,205,321
  321  IF(KPMTM(KSET))320,320,322
  322  J24  = KPMTM(KSET)
       GO TO (308,318),J24
  3..  CONTINUE
  1..    NTINUE
  33.  ONTINUE
       IF(NINCLD)501,501,510
  510  IF(K-1)501,501,511
  511  NINCPM = NINCPM - 1
       IF(NINCPM-3)513,515,515
  513  CONTINUE
```

```
      GO TO (501,512),NINCPM                                                0732
C     NINCSW = 1 MEANS THIS IS A RE-CYCLE STEP (NO LOAD OR TIME INCREASE)   0733
C     NINCSW = 2 MEANS THIS IS LAST RE-CYCLE AT THIS LOAD OR TIME LEVEL     0734
C     NINCSW = 3 MEANS THIS IS A LOAD OR TIME INCREMENT STEP                0735
  512 NINCSW = 2                                                            0736
      GO TO 514                                                             0737
  515 NINCSW = 1                                                            0738
      GO TO 514                                                             0739
  501 GO TO (502,503),J24                                                   0740
  502 PM   = PM   + DELPM                                                   0741
      GO TO 504                                                             0742
  503 TIME = TIME + DELTIM                                                  0743
  504 NINCPM = NINCLD + 2                                                   0744
      NINCSW = 3                                                            0745
  514 CONTINUE                                                              0746
      IF(NINCLD)524,524,525                                                 0747
  525 NINKCY = NINCLD + 3 - NINCPM                                          0748
      GO TO (523,524,524),NINCSW                                            0749
  523 NINKLU = NINKCY - (NINKCY/10)*10                                      0750
      IF(NINKLU)328,524,328                                                 0751
C     NINKCY IS COUNT OF TOTAL CYCLES AT THIS LOAD OR TIME LEVEL            0752
C     NINKLU = 0 WILL PRINT EACH TENTH CYCLE, IF THIS LOAD OR               0753
C     TIME LEVEL IS TO BE PRINTED                                           0754
  524 GO TO (324,325),J24                                                   0755
  324 CALL OUTPUT(PM,PMTM(KSET,3),KLU1)                                     0756
      GO TO 326                                                             0757
  325 CALL OUTPUT(TIME,PMTM(KSET,3),KLU1)                                   0758
  326 IF(KLU2)327,328,327                                                   0759
  327 KLU2 = KLU1                                                           0760
  328 CONTINUE                                                              0761
      IF(TIME)333,333,332                                                   0762
  332 GO TO(333,337),J24                                                    0763
C     COMPUTE REFERENCE CREEP TIME FOR ALL NODES                            0764
  337 DO 331 I3 = 1,NC                                                      0765
      EPCNP(I3) = EPCNK(I3)                                                 0766
      FACNUM = ABSF(EPCNP(I3))                                              0767
      EXPON  = BETA*ABSF(SGBARN(I3))                                        0768
      FACDN1 =( 2.71828183**EXPON)-1.                                       0769
      TIMK1(I3)  = (FACNUM/(ALPHA*FACDN1))**(1./GAMMA)                      0770
      GO TO (336,336,334),NINCSW                                            0771
  336 CONTINUE                                                              0772
      EPCNK(I3) = (ALPHA*(TIMK1(I3)        )**GAMMA)*FACDN1                 0773
      GO TO 331                                                             0774
  334 CONTINUE                                                              0775
      EPCNK(I3) = (ALPHA*(TIMK1(I3)+DELTIM)**GAMMA)*FACDN1                  0776
  331 CONTINUE                                                              0777
  333 CONTINUE                                                              0778
B 156 FA = 214747433125                                                     0779
      KLU9   = 1                                                            0780
      IF(KLU2)250,250,251                                                   0781
  250 CONTINUE                                                              0782
C     LABEL = APPLIED LOAD                                                  0783
B     FB = 246043462124                                                     0784
B     FC = 606060606060                                                     0785
      KLUSIZ = 2                                                            0786
      WRITE TAPE 8,KLU9,KLUSIZ,FA,FB,FC,PM,TIME                             0787
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  251 CONTINUE                                                          0788
      KLU9   = 2                                                        0789
      IF(KLU2+KLU8)252,252,253                                          0790
  252 CONTINUE                                                          0791
C     LABEL = REF. CREEP TIME                                           0792
B     FA = 512526336023                                                 0793
B     FB = 512525476063                                                 0794
B     FC = 314425606060                                                 0795
      WRITE TAPE 8,KLU9,     NC,FA,FB,FC,(TIMK1 (J1),J1=1,NC)           0796
B     FA = 252626336023                                                 0798
B     FB = 512525476062                                                 0799
B     FC = 635121314560                                                 0800
      WRITE TAPE 8,KLU9,     NC,FA,FB,FC,(EPCNK (J1),J1=1,NC)           0801
  253 CONTINUE                                                          0802
C     SAVE PRECEDING CYCLE VALUES OF EPBARN                             0803
      DO 152 I1 = 1,55                                                  0804
  152 EPBARP(I1) = EPBARN(I1)                                           0805
C     CALCULATE NODE STRESSES - MATRIX SIGUK - FRAME SIZE 165 X 1       0806
      DO 861 I5 = 1,NA                                                  0807
      SIGUK(I5) = SIM(I5)*PM                                            0808
      DO 861 I6 = 1,NA                                                  0809
      SIGUK(I5) = SIGUK(I5) + SIJ(I5,I6)*EPSUK(I6)                      0810
  861 CONTINUE                                                          0811
      IF(KLU2)254,254,255                                               0812
  254 CONTINUE                                                          0813
C     LABEL = NODE STRESSES                                             0814
B     FA = 454624256062                                                 0815
B     FB = 635125626225                                                 0816
B     FC = 626060606060                                                 0817
      WRITE TAPE 8,KLU9,     NA,FA,FB,FC,(SIGUK (J1),J1=1,NA)           0818
  255 CONTINUE                                                          0819
C     CALCULATE MAGNITUDE AND SIGN OF EFFECTIVE STRESS AT EACH NODE     0820
C     CALCULATE EFFECTIVE STRESSES - MATRIX SGBARN - FRAME SIZE 55 X 1  0821
      DO 166 I7 = 1,NC                                                  0822
      SGBARP(I7) = SGBARN(I7)                                           0823
      M3         = 3*I7                                                 0824
      M32        = M3-2                                                 0825
      M31        = M3-1                                                 0826
      SGBARN(I7)=SQRTF(AL1231(I7)*SIGUK(M32)**2-AL1212(I7)*SIGUK(M32)*  0827
     1SIGUK(M31)+AL1223(I7)*SIGUK(M31)**2+3.*ALFA44(I7)*SIGUK(M3)**2)   0828
  166 CONTINUE                                                          0829
      IF(KLU2)256,256,257                                               0830
  256 CONTINUE                                                          0831
C     LABEL = EFF. STRESSES                                             0832
B     FA = 252626336062                                                 0833
B     FB = 635125626225                                                 0834
B     FC = 626060606060                                                 0835
      WRITE TAPE 8,KLU9,     NC,FA,FB,FC,(SGBARN(J1),J1=1,NC)           0836
  257 CONTINUE                                                          0837
C     CALCULATE EFFECTIVE INELASTIC STRAIN FOR EACH NODE - INTERPOLATE  0838
C     IN TABLE (TSIGN VS. TEPSN)                                        0839
      DO 181 I8 = 1,NC                                                  0840
C     SGBARP IS EFFECTIVE STRESS OF PREVIOUS CYCLE                      0841
      IF(SGBARN(I8)-SGBARP(I8))411,401,401                              0842
C     EFFECTIVE STRESS IS ABOVE PREVIOUS LEVEL                          0843
C     SGBARM IS EFFECTIVE STRESS OF LAST CYCLE TO SHOW AN INCREASE      0844
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  401 IF(SGBARN(I8)-SGBARM(I8))411,402,402                                  0845
C     EFFECTIVE STRESS IS ABOVE KNEE OF PREVIOUS DROP-OFF, IF ANY          0846
  402 IF(SGBARP(I8)-SGBARM(I8))403,403,403                                  0847
  403 SGBARM(I8) = SGBARN(I8)                                               0848
      DO 171 I9 = 1,11                                                      0849
      ESUPRK =(SGBARN(I8)/E) + EPBARP(I8)                                   0850
      IF(ESUPRK-TEPSN(I9))173,172,171                                       0851
  171 CONTINUE                                                              0852
      GO TO 998                                                             0853
  172 BARSGN = TSIGN(I9)                                                    0854
      IF(SGBARN(I8)-TSIGN(2))176,177,177                                    0855
  177 AL1212(I8)= 2.*TALF12(I9)                                             0856
      AL1223(I8)= TALF12(I9)+ TALF23(I9)                                    0857
      AL1231(I8)= TALF12(I9)+ TALF31(I9)                                    0858
      ALFA44(I8)= TALF44(I9)                                                0859
  178 CONTINUE                                                              0860
      GO TO 174                                                             0861
  173 KKK2 = I9                                                             0862
      KKK1 = I9 - 1                                                         0863
      STNRAT    = (ESUPRK-TEPSN(KKK1))/(TEPSN(KKK2)-TEPSN(KKK1))            0864
      BARSGN    = TSIGN(KKK1)+(TSIGN(KKK2)-TSIGN(KKK1))*STNRAT              0865
      IF(SGBARN(I8)-TSIGN(2))176,175,175                                    0866
  175 CONTINUE                                                              0867
      ALFA12    = TALF12(KKK1)+(TALF12(KKK2)-TALF12(KKK1))*STNRAT           0868
      ALFA23    = TALF23(KKK1)+(TALF23(KKK2)-TALF23(KKK1))*STNRAT           0869
      ALFA31    = TALF31(KKK1)+(TALF31(KKK2)-TALF31(KKK1))*STNRAT           0870
      ALFA44(I8)=TALF44(KKK1)+(TALF44(KKK2)-TALF44(KKK1))*STNRAT            0871
      AL1212(I8)= 2.*ALFA12                                                 0872
      AL1223(I8)= ALFA12 + ALFA23                                           0873
      AL1231(I8)= ALFA12 + ALFA31                                           0874
  176 CONTINUE                                                              0875
      GO TO 174                                                             0876
  174 EPBARN(I8) = ESUPRK -  BARSGN / E                                     0877
C     CALCULATE TOTAL EFFECTIVE STRAIN - MATRIX TEFSTN - FRAME SIZE 55X1    0878
      TEFSTN(I8) = ESUPRK                                                   0879
  415 TEFSTN(I8) = TEFSTN(I8) + EPCNK(I8)                                   0880
C     CALCULATE EFFECTIVE STRAIN CHANGES - MATRIX DELEPN - FRAME 55 X 1     0881
C     CALCULATE INCREMENTAL EFFECTIVE INELASTIC STRAIN                      0882
      DELEPN(I8) = EPBARN(I8) - EPBARP(I8)                                  0883
      GO TO (420,425),J24                                                   0884
  425 DELEPN(I8) = DELEPN(I8) + EPCNK(I8) - EPCNP(I8)                       0885
  420 CONTINUE                                                              0886
      GO TO 181                                                             0887
C     DROP-OFF OF EFFECTIVE STRESS                                          0888
C     OR STILL BELOW THE KNEE OF PREVIOUS DROP-OFF                          0889
  411 EPBARN(I8) = EPBARP(I8)                                               0890
      TEFSTN(I8) = EPBARP(I8)+(SGBARN(I8)/E)                                0891
      TEFSTN(I8) = TEFSTN(I8) + EPCNK(I8)                                   0892
      DELEPN(I8) = 0.0                                                      0893
      GO TO (424,426),J24                                                   0894
  426 DELEPN(I8) = EPCNK(I8) - EPCNP(I8)                                    0895
  424 CONTINUE                                                              0896
      GO TO 181                                                             0897
  181 CONTINUE                                                              0898
      IF(KLU2+KLU8)266,266,267                                              0899
  266 CONTINUE                                                              0900
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C     LABEL = EFF.PLASTIC STRAIN                                        0901
8     FA = 252626334743                                                 0902
8     FB = 216263312360                                                 0903
8     FC = 626351213145                                                 0904
      WRITE TAPE 8,KLU9,     NC,FA,FB,FC,(EPBARN(J1),J1=1,NC)           0905
  267 CONTINUE                                                          0906
      IF(KLU2)258,258,259                                               0907
  258 CONTINUE                                                          0908
C     LABEL = TOTAL EFF. STRAIN                                         0909
8     FA = 634663214360                                                 0910
8     FB = 252626336062                                                 0911
8     FC = 635121314560                                                 0912
      WRITE TAPE 8,KLU9,     NC,FA,FB,FC,(TEFSTN(J1),J1=1,NC)           0913
  259 CONTINUE                                                          0914
      DO 862 I25=1,NC                                                   0915
  862 TEFSTN(I25) = 0.0                                                 0916
C     CALCULATE NODE STRAIN CHANGE - MATRIX DELEPK - FRAME SIZE 165 X 1  0917
      DO 191 I11 = 1,NC                                                 0918
      TEMPA =          DELEPN(I11)/ SGBARN(I11)                         0919
      M3       = 3*I11                                                  0920
      M32      = M3-2                                                   0921
      M31      = M3-1                                                   0922
      DELEPK(M32)=TEMPA*(AL1223(I11)*SIGUK(M32)-.5*AL1212(I11)*SIGUK(M31 0923
     1))                                                                0924
      DELEPK(M31)=TEMPA*(AL1223(I11)*SIGUK(M31)-.5*AL1212(I11)*SIGUK(M32 0925
     1))                                                                0926
      DELEPK(M3) =TEMPA*(3.*ALPA44(I11)*SIGUK(M3))                      0927
  191 CONTINUE                                                          0928
C     CALCULATE NODE PLASTIC STRAIN - MATRIX EPSUK - FRAME SIZE 165 X 1  0929
C     CALCULATE NODE POINT STRAINS                                      0930
      DO 192 I23=1,NA                                                   0931
  192 EPSUK (I23) = EPSUK (I23) + DELEPK(I23)                           0932
      IF(KLU2+KLU8)260,260,261                                          0933
  260 CONTINUE                                                          0934
C     LABEL = EFF.STRAIN CHANGES                                        0935
8     FA = 252626336263                                                 0936
8     FB = 512131456023                                                 0937
8     FC = 302145272562                                                 0938
      WRITE TAPE 8,KLU9,     NC,FA,FB,FC,(DELEPN(J1),J1=1,NC)           0939
C     LABEL = NODE STRAIN CHANGE                                        0940
8     FA = 454624256062                                                 0941
8     FB = 635121314560                                                 0942
8     FC = 233021452725                                                 0943
      WRITE TAPE 8,KLU9,     NA,FA,FB,FC,(DELEPK(J1),J1=1,NA)           0944
C     LABEL = NODE INELAS.STRAIN *                                      0945
8     FA = 454624256031                                                 0946
8     FB = 452543216233                                                 0947
8     FC = 626351213145                                                 0948
      WRITE TAPE 8,KLU9,     NA,FA,FB,FC,(EPSUK (J1),J1=1,NA)           0949
  261 CONTINUE                                                          0950
C     CALCULATE TOTAL NODE STRAINS - MATRIX TOTEPS - FRAME SIZE 165 X 1  0951
      DO 201 I14=1,NC                                                   0952
      M32 = 3*I14-2                                                     0953
      M31 = 3*I14-1                                                     0954
      M3  = 3*I14                                                       0955
      TOTEPS(M32)=EPSUK (M32)+SIGUK(M32)/E -GNU*SIGUK(M31)/E            0956
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      TOTEPS(M31)=EPSUK (M31)+SIGUK(M31)/E -GNU*SIGUK(M32)/E                0957
  201 TOTEPS(M3 )=EPSUK (M3 )+SIGUK(M3 )/SHRMOD                            0958
      IF(KLU2)262,262,263                                                  0959
  262 CONTINUE                                                             0960
C     LABEL = TOT. NODE STRAINS                                            0961
B     FA = 634663336045                                                    0962
B     FB = 462425606263                                                    0963
B     FC = 512131456260                                                    0964
      WRITE TAPE 8,KLU9,     NA,FA,FB,FC,(TOTEPS(J1),J1=1,NA)              0965
  263 CONTINUE                                                             0966
      IF(KLU2)268,268,269                                                  0967
  268 CONTINUE                                                             0968
      KLU9   = 3                                                           0969
      KLUSIZ = 2                                                           0970
      WRITE TAPE 8,KLU9,KLUSIZ, K,PM,PM,PM,TIME                            0971
      IF(NINCLD)269,269,273                                                0972
  273 CONTINUE                                                             0973
      KLU9   = 4                                                           0974
      KLUSIZ = 2                                                           0975
      WRITE TAPE 8,KLU9,KLUSIZ,PM,PM,PM,NINKCY,NINTOT                      0976
  269 CONTINUE                                                             0977
      IF(NINCLD)370,370,371                                                0978
  371 IF(NINCSW-2)422,370,422                                              0979
  370 CONTINUE                                                             0980
      IF(SENSE SWITCH 6)421,422                                            0981
  422 CONTINUE                                                             0982
      GO TO 151                                                            0983
  421 IF(KLU2)423,423,208                                                  0984
  423 CONTINUE                                                             0985
  205 KLU9   = 7                                                           0986
B     FA     = 777777777777                                              0987
      DO 206 I25 = 1,10                                                    0988
  206 WRITE TAPE 8,KLU9,KLU9,FA,FA,FA,FA,FA,FA,FA,FA,FA,FA                 0989
      END FILE 8                                                           0990
      END FILE 8                                                           0991
      END FILE 8                                                           0992
      END FILE 8                                                           0993
      END FILE 8                                                           0994
      END FILE 8                                                           0995
      REWIND 8                                                             0996
      IF(SENSE SWITCH 6)207,209                                            0997
  209 IF(J24-3)210,207,210                                                 0998
  207 CONTINUE                                                             0999
      PAUSE 1                                                              1000
      REWIND 11                                                            1001
      CALL CHAIN(5,3)                                                      1002
  210 CONTINUE                                                             1003
      CALL CHAIN(6,3)                                                      1004
  208 KERRSW = 2                                                           1005
C     KERRSW SET TO 2 TO MAKE KLU2 = 0 AND PRINT A CYCLE                   1006
      GO TO 151                                                            1007
  998 KLU9   = 6                                                           1008
      KLUSIZ = 3                                                           1009
      WRITE TAPE 8,KLU9,KLUSIZ, K,I8,PM,ESUPRK,SGBARN(I8),TIME             1010
      GO TO 205                                                            1011
      END(1,1,0,0,0,0,1,1,0,1,0,0,0,0,0)

*     LIST8                                                                1013
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      SUBROUTINE OUTPUT(VALUE1,STEP1,KLU1)                              1014
C     THIS SUBROUTINE SETS KLU1 = 0 IF THE CURRENT CYCLE IS TO BE PRINTED 1015
      VALUE = ABSF(VALUE1)                                              1016
      STEP  = ABSF(STEP1)                                               1017
  102 IF(VALUE-STEP)131,100,100                                         1018
  100 NTEST1 = (VALUE/STEP)*1.00001                                     1019
      NTEST2 = (VALUE/STEP)+.998                                        1020
      IF(NTEST1-NTEST2)131,130,131                                      1021
  130 KLU1   = 0                                                        1022
      GO TO 135                                                         1023
  131 KLU1   = 1                                                        1024
  135 RETURN                                                            1025
      END(1,1,0,0,0,0,0,1,1,0,1,0,0,0,0,0)

*     CHAIN(5,3)                                                        1027
*     LIST8                                                             1028
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451285  GRUMMAN AIRCRAFT ENGINEERING CORP.   DECK NO. 45128   LINK 5

C451285  MATRIX ANALYSIS OF INELASTIC PLATE - LINK 5 - WITH CREEP                1030
C       THIS LINK WRITES A SAVE TAPE FOR RESTART                                1031
        COMMON TEFSTN, TOTEPS                                                   1032
        COMMON   KLU4,   KLU5,   KLU6,   KLU8,     NA,     NC,      K          1033
        COMMON KERRSW, NINCLD,   KSET,     PM,      E,    GNU, SHRMOD          1034
        COMMON  ALPHA,   BETA,  GAMMA,   TIME, KLUISO                           1035
        COMMON   PMTM,  KPMTM,    SIM,    SIJ,  TSIGN,   TEPSN                  1036
        COMMON TALF12, TALF23, TALF31, TALF44                                   1037
        COMMON AL1212, AL1223, AL1231, ALFA44                                   1038
        DIMENSION TALF12(11),TALF23(11),TALF31(11),TALF44(11)                   1039
        DIMENSION AL1212(55),AL1223(55),AL1231(55),ALFA44(55)                   1040
        COMMON  SIGUK,  EPSUK                                                   1041
        COMMON SGBARN, SGBARP, SGBARM, EPBARN, EPBARP, DELEPN                   1042
        COMMON  EPCNK,  EPCNP                                                   1043
        EQUIVALENCE (TIMK1,TEFSTN),(TOTEPS,DELEPK)                              1044
C       THESE FOUR ARRAYS ARE EQUIVALENCED TO SAVE CORE SPACE.                  1045
        DIMENSION TIMK1(55),TEFSTN(55),TOTEPS(165),DELEPK(165)                  1046
        DIMENSION PMTM(10,3),KPMTM(10),SIM(165),SIJ(165,165),TSIGN(11)          1047
        DIMENSION TEPSN(11),SIGUK(165),EPSUK(165),SGBARN(55),SGBARP(55)         1048
        DIMENSION SGBARM(55),EPBARN(55),EPBARP(55),DELEPN(55),EPCNK(55)         1049
        DIMENSION EPCNP(55)                                                     1050
        REWIND 11                                                               1051
        WRITE OUTPUT TAPE 11,33                                                 1052
        END FILE 11                                                             1053
        WRITE TAPE 11,KLU4,KLU5,KLU6,KLU8,NA,NC,K,KERRSW,NINCLD,KSET,PM,E,      1054
       1GNU,SHRMOD,ALPHA,BETA,GAMMA,TIME,KLUISO                                 1055
        WRITE TAPE 11,PMTM                                                      1056
        WRITE TAPE 11,KPMTM                                                     1057
        WRITE TAPE 11,SIM                                                       1058
        WRITE TAPE 11,SIJ                                                       1059
        WRITE TAPE 11,TSIGN                                                     1060
        WRITE TAPE 11,TEPSN                                                     1061
        WRITE TAPE 11,TALF12                                                    1062
        WRITE TAPE 11,TALF23                                                    1063
        WRITE TAPE 11,TALF31                                                    1064
        WRITE TAPE 11,TALF44                                                    1065
        WRITE TAPE 11,AL1212                                                    1066
        WRITE TAPE 11,AL1223                                                    1067
        WRITE TAPE 11,AL1231                                                    1068
        WRITE TAPE 11,ALFA44                                                    1069
        WRITE TAPE 11,SIGUK                                                     1070
        WRITE TAPE 11,EPSUK                                                     1071
        WRITE TAPE 11,SGBARN                                                    1072
        WRITE TAPE 11,SGBARP                                                    1073
        WRITE TAPE 11,SGBARM                                                    1074
        WRITE TAPE 11,EPBARN                                                    1075
        WRITE TAPE 11,EPBARP                                                    1076
        WRITE TAPE 11,DELEPN                                                    1077
        WRITE TAPE 11,EPCNK                                                     1078
        WRITE TAPE 11,EPCNP                                                     1079
        END FILE 11                                                             1080
        END FILE 11                                                             1081
        END FILE 11                                                             1082
C       SUBROUTINE RUN REWINDS AND UNLOADS THE DESIGNATED TAPE                  1083
        CALL RUN (11)                                                           1084

        PRINT 32                                                                1085
        KERRSW = 5                                                              1086
        CALL CHAIN(6,3)                                                         1087
   32 FORMAT(//52H * * * SAVE TAPE A-6 FOR RESTART AT THIS POINT * * *)         1088
   33 FORMAT(67HCONTIN    99 SAVE THIS TAPE FOR RERUN DECK 45128 INELAST        1089
     1IC ANALYSIS )                                                             1090
        END(1,1,0,0,0,0,0,1,1,0,1,0,0,0,0,0)


*       CHAIN(6,3)                                                              1092
*       LIST8                                                                   1093
```

```
451286  GRUMMAN AIRCRAFT ENGINEERING CORP.   DECK NO. 45128   LINK 6
C451286  MATRIX ANALYSIS OF INELASTIC PLATE - LINK 6 - WITH CREEP
C      THIS LINK CONVERTS BINARY OUTPUT ON TAPE 8 TO BCD
C      ON THE MONITOR PRINT TAPE
       COMMON TEFSTN, TOTEPS
       COMMON   KLU4,   KLU5,   KLU6,   KLU8,     NA,     NC,     K
       COMMON KERRSW, NINCLD,   KSET,     PM,      E,    GNU, SHRMOD
       COMMON  ALPHA,   BETA,  GAMMA,   TIME, KLUISO
       DIMENSION TEFSTN(55),TOTEPS(105)
       DIMENSION ARRAY(165),LIST(105)
       EQUIVALENCE (FA,IFA),(FB,IFB),(FC,IFC),(ARRAY,LIST)
       REWIND 8
  101  READ TAPE 8,KLU9,KLUSIZ,FA,FB,FC,(ARRAY(J1),J1=1,KLUSIZ)
       IF(KLU9)101,101,106
  106  IF(KLU9-9)111,111,101
  111  GO TO(121,122,123,124,125,126,127,128,129),KLU9
  121  CONTINUE
       WRITE OUTPUT TAPE 6,22,FA,FB,FC,ARRAY(1),ARRAY(2)
       GO TO 101
  122  CONTINUE
       WRITE OUTPUT TAPE 6,21,FA,FB,FC,(ARRAY(J1),J1=1,KLUSIZ)
       GO TO 101
  123  CONTINUE
       WRITE OUTPUT TAPE 6,31,IFA,ARRAY(1),ARRAY(2)
       GO TO 101
  124  CONTINUE
       WRITE OUTPUT TAPE 6,32,LIST(1),LIST(2)
       GO TO 101
  126  CONTINUE
       WRITE OUTPUT TAPE 6,12,ARRAY(1),ARRAY(2)
       WRITE OUTPUT TAPE 6,13,IFA,IFB,FC,ARRAY(3)
       GO TO 101
  127  CONTINUE
       PRINT 14
       WRITE OUTPUT TAPE 6,14
       REWIND 8
       IF(KERRSW-5)132,131,132
  131  PRINT 33
  132  CALL EXIT
  125  CONTINUE
  128  CONTINUE
  129  CONTINUE
       GO TO 101
   12  FORMAT(46H VALUE NOT FOUND IN TABLE FOR EPSILON BAR N = ,E15.8,
      119H   ( SIGMA BAR N = ,E15.8,2H ) )
   13  FORMAT(16H CYCLE NUMBER = ,I5,20H    ELEMENT INDEX = ,I4,17H    LO
      1AD LEVEL = ,F9.2,11H    TIME = ,F8.4)
   14  FORMAT(76H DO NOT SAVE TAPE 8-1 - IT HAS BEEN COMPLETELY PROCESSED
      1 ONTO THE PRINT TAPE)
   21  FORMAT(// 1X,3A6,5(1PE16.7)/(19X,5E16.7))
   22  FORMAT(//1X,3A6,2X,F12.2,5X,6HTIME =,F12.6)
   31  FORMAT(2HO ,I6,27H CYCLES COMPLETED - LOAD = ,F 9.2,
      1          12H     TIME = ,F 8.4)
   32  FORMAT(1H+,68X,6HCYCLE ,I3,4H OF ,I3,27H AT THIS LOAD OR TIME LEVE
      1L)
   33  FORMAT(//52H * * * SAVE TAPE A-8 FOR RESTART AT THIS POINT * * *)

       END(1,1,0,0,0,0,1,1,0,1,0,0,0,0,0)

*      DATA
```

APPENDIX VI

STRESS DISTRIBUTIONS DUE TO UNIT INITIAL STRAINS

The basic matrix utilized in the nonlinear and inelastic analysis described in this report is the initial strain influence coefficient matrix $[\Gamma_{uv}]$. Elements of this matrix provide the u^{th} stress component caused by a unit initial strain at the v^{th} stress location. The first description and derivation of this useful matrix was made by Denke in 1954, Reference 20. The matrix is generated by a structural analysis which may be of the force or stiffness type.

A. Force Method Application

The method utilized here is a simple extension of redundant structure analysis. Essentially, the additional work involved is the calculation of displacements at the applied loads and redundants in the statically determinant structure caused by initial strains. These displacements are combined with those caused by the applied loads and redundants. The final step is to adjust the magnitude of the redundants to eliminate the total displacements at the redundants. From this point the determination of stress distributions for the redundant structure is carried out as before.

The displacement in the statically determinate structure at the t^{th} applied load or redundant due to initial strain can be expressed by use of the principle of virtual work as follows:

$$\alpha_{tv} = \sum_{a} \int_{\substack{\text{Volume of} \\ a^{th} \text{ member}}} (\sigma_{xt} \cdot \epsilon_{xv}) dV \qquad \text{(A-1)}$$

In the above expression, α_{tv} is the displacement at the t^{th} unit applied load or redundant due to the v^{th} initial strain, σ_{xt} is the direct stress in the a^{th} member due to the unit load, and ϵ_{xv} is the initial strain in the a^{th} member at the corresponding stress point. The summation $\sum_{a}$ indicates that the virtual work in all members affected by the induced strain, ϵ_{xv}, must be considered.

Equation A-1 is written for the uniaxial direct stress condition that is commonly assumed to exist in bar members as pictured in Figure (1). The effect of a shear panel has been omitted in the derivation for simplicity. Terms necessary for inclusion of shear may be derived in a similar manner.

Suppose that the a^{th} member of a structure is a bar, with cross sectional area A, linearly varying axial load $\sigma_{xt} \cdot A$, and linearly varying initial strain ϵ_{xv}.

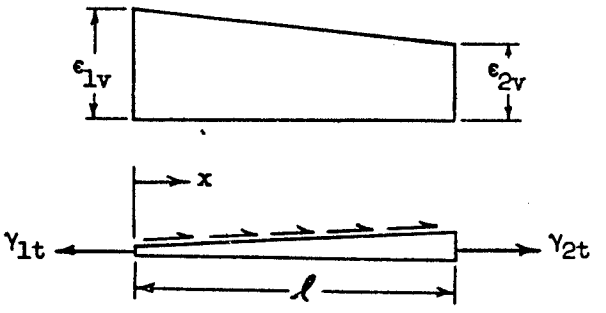

Figure 1

Then

$$\sigma_{xt} = \frac{1}{A}\left\{\gamma_{1t}\left(1 - \frac{x}{\ell}\right) + \gamma_{2t}\frac{x}{\ell}\right\}$$

$$\epsilon_{xv} = \epsilon_{1v}\left(1 - \frac{x}{\ell}\right) + \epsilon_{2v}\frac{x}{\ell}$$

and after evaluation

$$\int_{\substack{\text{Volume of} \\ a^{th} \text{ member}}} \sigma_{xt} \cdot \epsilon_{xv} dV = [\gamma_{1t}\ \gamma_{2t}] \begin{bmatrix} L_{a_{11}} & L_{a_{12}} \\ L_{a_{21}} & L_{a_{22}} \end{bmatrix} \begin{Bmatrix} \epsilon_{1v} \\ \epsilon_{2v} \end{Bmatrix}$$

where

$$L_{a_{11}} = L_{a_{22}} = \frac{\ell}{3}$$
$$L_{a_{12}} \quad L_{a_{21}} = \frac{\ell}{6}$$

The $L_{a_{ij}}$ matrix can be similarly determined for other types of structural elements. For rectangular shear panels, of dimensions b and h, it can be shown that the $L_{a_{ij}}$ factor for shear strain γ_{xy}, is bh, the panel flat plate area. For nonorthogonal structure (swept panels) other $L_{a_{ij}}$ factors may be developed. The sum of such matrices for the entire structure is designated $[L_{ij}]$.

The matrix expression for the displacements at all of the applied loads and redundants due to unit initial strain is

$$\left[\begin{array}{c} \alpha_{mv} \\ \hline \alpha_{rv} \end{array}\right] = \left[\gamma_{im} \mid \gamma_{ir}\right]^{T}\left[L_{iv}\right] \qquad \text{A-2}$$

The ϵ_v's are the initial strains corresponding to each of the member loads q_v.

Utilizing Equation A-2, the redundants are evaluated by

$$\left[\alpha_{rm} \mid \alpha_{rv} \mid \alpha_{rs}\right]\left\{\begin{array}{c} P_m \\ \epsilon_v \\ q_s \end{array}\right\} = 0$$

the solution of which is

$$\left\{q_s\right\} = \left[\Gamma_{sm} \mid \Gamma_{sv}\right]\left\{\frac{P_m}{\epsilon_v}\right\}$$

where

$$\left[\Gamma_{sm} \mid \Gamma_{sv}\right] = -\left[\alpha_{rs}\right]^{-1}\left[\alpha_{rm} \mid \alpha_{rv}\right]$$

The member loads in the redundant structure become

$$\left\{q_i\right\} = \left[\Gamma_{im} \mid \Gamma_{iv}\right]\left\{\frac{P_m}{\epsilon_v}\right\} \qquad \text{A-3}$$

where

$$\left[\Gamma_{im} \mid \Gamma_{iv}\right] = \left[\gamma_{im} \mid \gamma_{ir}\right]\left[\begin{array}{c|c} I & 0 \\ \hline \Gamma_{rm} & \Gamma_{rv} \end{array}\right]$$

Member stresses are obtained by pre-multiplying member loads by $[\beta_{ui}]$, the reciprocal values of appropriate bar areas and skin gages.

$$\left[\Gamma_{um} \mid \Gamma_{uv}\right] = \left[\beta_{ui}\right]\left[\Gamma_{im} \mid \Gamma_{iv}\right]$$

then

$$\left\{\sigma_u\right\} = \left[\Gamma_{um} \mid \Gamma_{uv}\right]\left\{\frac{P_m}{\epsilon_v}\right\} \qquad \text{A-4}$$

Digital computer programs which are available for conventional force method analyses may be used to determine the Γ_{um}, Γ_{uv} matrices. This is readily accomplished by redefining several input matrices.

(1) replace $[\gamma_{im}]$, the usual unit load distribution in the statically determinate structure due to applied loads by

$$\left[\begin{array}{c|c} \gamma_{im} & 0 \\ \hline 0 & I \end{array}\right]_{(2i)\ x\ (m\ +\ i)}$$

the unit diagonal matrix I has as many elements as there are member loads in the structure.

(2) replace $[\gamma_{ir}]$, the usual unit load distribution in the statically determinate structure due to redundants by

$$\left[\begin{array}{c} \gamma_{ir} \\ \hline 0 \end{array}\right]_{2i\ x\ r}$$

(3) replace $[\alpha_{ij}]$, the member flexibility matrix by

$$\left[\begin{array}{c|c} \alpha_{ij} & L_{iv} \\ \hline L_{iv} & 0 \end{array}\right]_{2i\ x\ 2i}$$

The L_{iv} values are the geometrical factors of Equation A-2. Straightforward matrix operation will now yield a load distribution matrix for the redundant structure which may be identified as follows:

$$\left[\begin{array}{c|c} \Gamma_{im} & \Gamma_{iv} \\ \hline 0 & I \end{array}\right]$$

The upper portion $[\Gamma_{im} \mid \Gamma_{iv}]$ are the values defined by Equation A-3, and stresses may be obtained by using Equation A-4.

B. Direct Stiffness Method

Consider a truss shown in Figure (2a) where all the nodes are "locked" (prevented from displacing). If a strain ϵ_o is induced in a particular member it will produce a stress in only that member equal to $\sigma_o = - E\epsilon_o$ since all the nodes are locked. This stress requires node forces as shown in Figure (2b). The negatives of these node forces are applied to the "unlocked" structure and the stresses in all the members, (Figure 2c) are computed. The final stress will be the superposition of the "locked in" stresses and the stresses produced by the node forces acting on the unlocked structure. That is

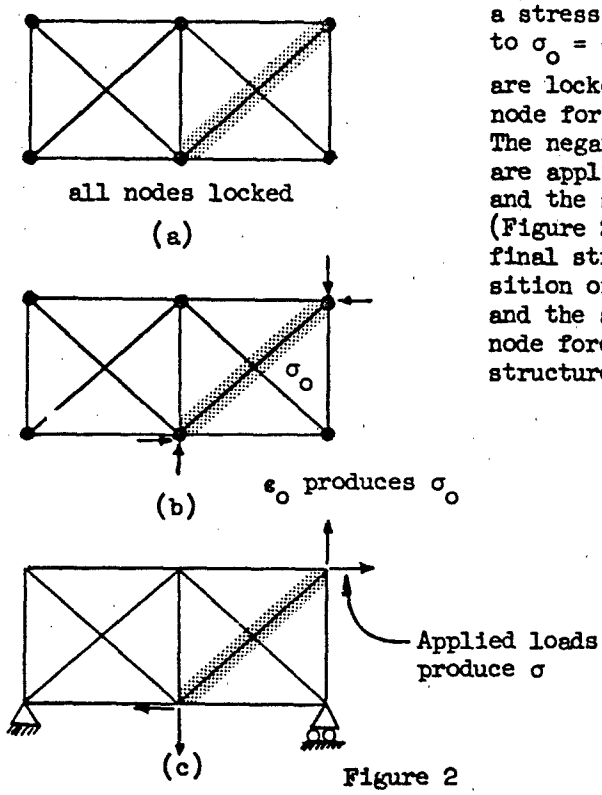

Figure 2

$$\sigma_T = \sigma + \sigma_o$$

Derivation of $[\Gamma_{um}]$ and $[\Gamma_{uv}]$

Consider an element of the total structure, let this element be supported in a statically determinate fashion. The total strain ϵ is then written as the sum of two strains namely ϵ_σ - the strains due to stresses, and ϵ_o - a set of induced strains. Thus

$$\{\epsilon\} = \{\epsilon_\sigma\} + \{\epsilon_o\} \qquad \text{B-1}$$

Rewriting this,

$$\{\epsilon_\sigma\} = \{\epsilon\} - \{\epsilon_o\} \qquad \text{B-2}$$

The stresses may be expressed in terms of the strains by using Hooke's law,

$$\{\sigma\} = [b]\{\epsilon_\sigma\} \qquad \text{B-3}$$

Thus

$$\{\sigma\} = [b]\{\epsilon\} - [b]\{\epsilon_o\} \qquad \text{B-4}$$

$\{\epsilon\}$ (the total strains) may be expressed in terms of the displacements of the nodes that affect the particular member in question. That is

$$\{\epsilon\} = [a]\{\delta\} \qquad \text{B-5}$$

It should be noted that the expression contains the basic assumption governing the behavior of the element. For example, for a triangle or bar element it will contain the assumption that the strains are constant throughout the element. Substituting this into Equation B-4

$$\{\sigma\} = [b][a]\{\delta\} - [b]\{\epsilon_o\} \qquad \text{B-6a}$$

$$= [s_d]\{\delta\} - [b]\{\epsilon_o\} \qquad \text{B-6b}$$

From virtual work it can be shown that the nodal forces $\{f\}$, associated with $\{\delta\}$, may be expressed in terms of the stresses by:

$$\{f\} = \int_{Vol} [a]^T\{\sigma\}dV \qquad \text{B-7}$$

Substituting B-6a into B-7 yields:

$$\{f\} = \int_{Vol} [a]^T[b][a]\{\delta\}dV - \int_{Vol} [a]^T[b]\{\epsilon_o\}dV \qquad \text{B-8}$$

For simple elements such as bars or triangles where the strains are assumed to be constant over the element none of the matrices within the integrals will be functions of the coordinates x, y or z and hence they are independent of the integration. The first portion of (B-8) yields the standard stiffness matrix [k] while the second portion yields the nodal forces due to induced strains $\{\epsilon_o\}$.

$$\{f\} = [k]\{\delta\} - V[a]^T[b]\{\epsilon_o\} \qquad \text{B-9}$$

In this expression, V is the volume of the element. Noting that $[a]^T[b] = \left[[b][a]\right]^T = [s_d]^T$ yields

$$\{f\} = [k]\{\delta\} - V[s_d]^T\{\epsilon_o\} \qquad \text{B-10}$$

The total stiffness matrix for the entire structure is obtained by superposing the stiffnesses for the individual elements to yield:

$$\{F\} = [K]\{\Delta\} - [S_d]^T[V]\{\epsilon_{oT}\} \qquad \text{B-11}$$

Where $\{F\}$ represents all of the external nodal forces, $\{\Delta\}$ represents all of the nodal displacements, $\{\epsilon_{oT}\}$ represents a column of all the induced strain components and $[V]$ is a diagonal matrix containing the values of the volumes of the various elements. (Note that for a triangle, where three strain components may be induced, the volume will appear three times). $[S_d]$ is the total stress matrix array which is obtained by proper arrangement of the individual stress matrices $[s_d]$.

Applying boundary conditions (via matrix [BC]) to equation (B-11) and introducing the applied external loads $\{P\}$, with nodal distribution $[L]$, yields:

$$[L]\{P\} = [BC]^T[K][BC]\{\Delta'\} - [BC]^T[S_d]^T[V]\{\epsilon_{oT}\} \qquad \text{B-12}$$

which may be solved to give the allowed nodal displacements under the boundary conditions:

$$\{\Delta'\} = [K_{11}]^{-1}[L]\{P\} + [K_{11}]^{-1}[BC]^T[S_d]^T[V]\{\epsilon_{oT}\} \qquad \text{B-13}$$

where

$$[K_{11}] = [BC]^T[K][BC]$$

For the total structure, B-6b may be written as:

$$\{\sigma_T\} = [S_d]\{\Delta\} - [B]\{\epsilon_{oT}\} \qquad \text{B-14}$$

where $[B]$ is a diagonal block of the elastic relations $[b]$ for each member. Substituting (B-13) into (B-14) and using the relation $\{\Delta\} = [BC]\{\Delta'\}$ yields:

$$\{\sigma_T\} = [S_d][BC][K_{11}]^{-1}[L]\{P\}$$

$$+ \left[[S_d][BC][K_{11}]^{-1}[BC]^T[S_d]^T[V] - [B]\right]\{e_{\sigma T}\} \qquad \text{B-15}$$

which, to use the previous notation, may be written as:

$$\{\sigma_u\} = [\Gamma_{um}]\{P_m\} + [\Gamma_{uv}]\{e_v\} \qquad \text{B-16}$$

The first matrix, $[\Gamma_{um}]$, of this expression, is the conventional distribution obtained for unit applied loads as indicated in Section A of this appendix.

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13. ABSTRACT

Most aerospace structural materials exhibit some degree of anisotropic strain hardening. A measure of anisotropy is introduced into structural elements by such fabrication processes as drawing, cold rolling and extrusion. During the past few years, several methods have appeared in the literature for introducing inelastic isotropic material behavior effects into existing matrix analysis routines. A review is presented of one of these methods. It is essentially a step-by-step calculation procedure, and corresponds to the flow theory of plasticity. The method has been extended to include the effects of anisotropic material and is formulated as a standard initial strain influence coefficient problem. Several analyses of an aluminum alloy (2024-T4) shear lag structure which has been tested previously for the Air Force are carried out first assuming isotropic material properties and then anisotropic properties. The resulting correlation between test results and that predicted by isotropic theory is reasonably good.

An analysis of a 1100F aluminum shear lag structure, carried out by the incremental method, gave reasonably good agreement. However, the anisotropic creep capability was not checked for want of test data. The approach is a reasonably good phenomenological model of a complex physical problem. The digital computer program submitted is suited for inclusion of other material nonlinearity.

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